A Study of Umbrella Damages from Bid-Rigging

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Abstract

If non-cartel firms adjust their pricing to the supra-competitive level sustained by a cartel, purchasers from non-cartel firms may suffer umbrella damages. This paper examines the bidding behavior of non-cartel firms against the Texas school milk cartel between 1980 and 1992. Evidence is found that the largest non-cartel firm bid less aggressively when facing the cartel. Structural estimation reveals that, per contract, damages due to non-cartel firms bidding higher are at least 35% of damages caused by the cartel. Inefficiencies raise the winner’s cost by 5.9%. These results shed light on the importance of umbrella damages from a civil liability perspective.

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1 Introduction

When a cartel does not include every firm competing in an industry, non-cartel firms can set their prices higher than they would otherwise have been able to under competitive conditions. This occurs, in particular, in markets where contracts are awarded via first-price procurement auctions. When the results of such procurement procedures (bids and bidders identities) are not concealed from non-cartel firms, they may serve as an indication of the prevailing price level when future contracts are procured for. Consequently, non-cartel bidders benefit from the protection of the cartel’s inflated bidding, and operate “under the cartel’s umbrella.” Purchasers from non-cartel bidders will still pay a price that exceeds what the market price would be in the absence of collusion. In this sense, damages inflicted by non-cartel bidders broaden the scope of cartel damages. Nonetheless, empirical research investigating the importance of such damages remains scarce.\footnote{This paper conducts a detailed study of umbrella damages by examining the bidding behavior of non-cartel bidders facing the Texas school milk cartel between 1980 and 1992.}

Bid-rigging was common in auctions for the supply of milk to schools, at least until the early 1990s. According to Porter and Zona (1999), investigations were conducted in more than twenty states across the U.S., more than $90 million of fines were imposed, while about 90 people were sent to jail for sentences lasting six months on average. The Texas milk cartel is well-suited for analyzing damages inflicted by non-cartel bidders. First, two essential conditions are met: the cartel was not all-inclusive, and the auction format was first-price sealed bid.\footnote{In ascending or second price auctions, bidding one’s private valuation is a dominant strategy irrespective of the existence of the cartel, therefore, umbrella damages do not arise. For umbrella damages to arise in first-price procurement auction, non-cartel firms need not know the existence of the cartel per se, only knowledge of the equilibrium distribution of their lowest competing bid is required to bid optimally.} In particular, bids and bidder identities are publicly announced. This public information enables non-cartel firms to learn and adjust their bids to the supra-competitive levels sustained by the cartel. Second, all firms involved in the cartel were convicted, which allows me to isolate non-cartel bidders and focus on their bidding behavior. Third, the dataset collected by the Antitrust Division of the Department of Justice spans markets with and without cartel operations: as such, it provides a unique opportunity to document how non-cartel firms’ bidding behavior is affected by the cartel’s presence.\footnote{In using data from both competitive and rigged markets, I follow previous empirical studies of price-fixing, e.g., Clark and Houde (2014) and Clark, Coviello, and Shneyerov (2018).}

Reduced form analysis of the bid data reveals that the largest non-cartel firm bid sig-
nificantly less aggressively when facing the cartel, controlling for auction and bidder-specific heterogeneity. Further investigation of cartel and umbrella damages and inefficiencies requires the estimation of a structural model of bidding in first-price auctions. Economic damages to the auctioneer are decomposed into (1) cartel damages, defined as the difference between the actual price paid to the cartel and the counterfactual competitive price that would have existed in the market had there not been a conspiracy and (2) umbrella damages, defined as the difference between the actual price paid to the non-cartel firm winning against the cartel and the counterfactual competitive price.

Because the cartel’s internal structure is unknown, damages are derived assuming two cases for the cartel mechanism. Either the mechanism is efficient (i.e., the lowest cost cartel member is the cartel bidder in the target auction), or the mechanism is inefficient (i.e., the cartel bidder is selected randomly among cartel members). I show formally that the efficient and inefficient mechanisms provide bounds on damages within a simple yet rich class of cartel mechanisms.

The structural analysis shows that, per contract, umbrella damages (conditional on the non-cartel firm winning) are at least 35% of cartel damages (conditional on the cartel winning). This lower bound is obtained with an efficient cartel. If the cartel is inefficient, umbrella damages can be as large as cartel damages. Conditional on the non-cartel firm winning, prices are inflated by 1.6% to 6.4% relative to the competitive winning bid. These damages are consistent with the reduced form estimates of a 6.2% overcharge, as well as the overcharge found by Porter and Zona (1999) for the Ohio school milk market. The results point to a cartel mechanism that was not entirely efficient, but far from inefficient.

In this particular setting, partial collusion exacerbates asymmetry among bidders in the first-price procurement: the cartel bidder has a stronger incentive to inflate its bid above its cost than the non-cartel bidder. As a result, the winner is not necessarily the lowest cost bidder and the auction is no longer efficient. Inefficiencies due to the asymmetry between non-cartel and cartel bidders raise the winner’s cost by 5.9% or $2,337 per contract.

The results are robust to the inclusion of endogenous entry by non-cartel firms in rigged districts. Indeed, high collusive prices and reduced competition may have attracted entry by non-cartel firms. A comprehensive damage assessment (in particular, when simulating

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4 Whereas the definition of an efficient cartel mechanism is straightforward, it is less evident for an inefficient cartel mechanism. In the inefficient case, the cartel bidder could be randomly selected as is assumed in this paper. However, one could think of other inefficient mechanisms such as the one in which the cartel bidder is the least cost-efficient member.

5 This is consistent with Pesendorfer (2000) who finds that, although the cartel did not use side-payments, it managed to retain quasi-efficient collusive rents through market division.

6 Demand for milk by schools is inelastic, therefore, any surplus loss stems from the possibility that the cartel loses when it is the lowest cost provider.
the counterfactual competitive winning bids) should account for the possibility that non-cartel firms may not have participated absent collusion. Damage estimates are smaller when accounting for endogenous entry, but umbrella damages still constitute a significant and similar fraction of cartel damages as in the baseline model with exogenous entry. The counterfactual results are also robust to the inclusion of ex-ante asymmetries between firms (e.g., plant to school district distances, firm capacities) and the use of alternative approaches to control for auction heterogeneity.

From a competition law perspective, U.S. and European civil courts have historically not recognized the ability of purchasers from non-cartel firms to pursue colluders for umbrella damages, the reason being the complexity of proving umbrella claims. This paper sheds new light on this debate by providing a case study of umbrella damages, emphasizing the type of data and methodology that renders the estimation of damages possible and far from speculative. The U.S. and European competition laws have recently taken divergent paths in their treatment of whether the civil liability in damages of the cartel members extends to umbrella damages. On June 5th 2014, the European Court of Justice (ECJ) handed down the awaited judgment in the elevator cartel case (*Kone AG and Others v. ÖBB-Infrastruktur AG*), stating the right of plaintiffs to compensation for umbrella damages. In its press release No 79/14, the ECJ states:

> [...] where it has been established that the cartel is, in the circumstances of the case and, in particular, the specific aspects of the relevant market, liable to result in prices being raised by competitors not a party to the cartel, the victims of this price increase must be able to claim compensation for loss sustained from the members of the cartel.

At the same time, the ECJ emphasizes the “high hurdles in terms of the burden of proof that await” any umbrella claimants.

In the U.S., competition law is inconsistent in its standing vis-a-vis umbrella claims. The ability of purchasers from non-cartel firms to recover damages from the conspirators under section 4 of the Clayton Act is uncertain because of the Supreme Court decision in *Illinois Brick Co. v. Illinois*. In the case, the Court ruled that indirect purchasers (downstream buyers) may not sue on a theory that a price-fixing overcharge has been passed on to them.

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7 The elevator cartel, which involved anticompetitive agreements between major European manufacturers of elevators and escalators (Kone, Otis, Schindler, and ThyssenKrupp) operated in several European Union states over many years. The European Commission uncovered the cartel in 2003 and, in 2007, imposed fines for the elevator cartel’s practices in the Belgian, German, Netherlands and Luxembourg markets.

8 Opinion of the Advocate General Kokott.

9 Section 4 of the Clayton Act provides that “any person ... injured in his business or property” by an antitrust violation may bring an action for treble damages.
by intermediate sellers purchasing from upstream colluders. This case was relied on by courts to deny standing to purchasers from non-cartel firms.\textsuperscript{10} Again, one of the policy reasons underlying the \textit{Illinois Brick} doctrine revolves around the complexity of tracing out the causal link between the antitrust violation and non-cartel firms’ response.\textsuperscript{11} This paper shows that in the particular case of bid-rigging of procurement auctions, bid data can be leveraged to estimate the size of damages when buyers do not contract directly with colluders. Moreover, accounting for non-cartel firms’ best-response to the cartel is not only necessary to assess umbrella damages, but is in fact crucial to getting a coherent analysis of the (direct) cartel damages inflicted. A partial equilibrium analysis, holding non-cartel firms’ bid distribution fixed to its pre-collusion level, would most likely result in incorrect damage assessments.\textsuperscript{12}

This paper also relates to the debate around auction format choice. A commonly advanced argument in favor of sealed bid auctions is that open auctions are more prone to collusion because conspirators can immediately punish any deviations.\textsuperscript{13} This reasoning does not account for the fact that given (partial) collusion, the auctioneer will suffer damages of a greater scope (measured by the number of auctions with an inflated winning bid) in sealed bid auctions: as non-cartel firms adjust their bidding strategy to the cartel, damages to the auctioneer extend to contract won by non-conspiring firms. Whereas in ascending auctions, the auctioneer suffers damages only when the cartel can suppress the second highest bid (the auctioneer may benefit in some cases from the cartel’s overbidding as found by Asker (2010)). This work shows that a buyer from non-conspiring firms can suffer damages of a similar magnitude than if they contracted directly with conspiring firms. The potential merit of sealed bid relative to open auctions is therefore nuanced with regard to these findings.


\textsuperscript{11}Many states, including California, have enacted Illinois Brick-repealer legislation providing indirect purchasers standing to sue for antitrust violations.

\textsuperscript{12}See Section 2 and footnote 2 for a detailed discussion of why the cartel’s existence need not be common knowledge for non-cartel firms to best-respond. For the Texas school milk cases, the criminal cases documents are available and include information about penalties to colluders. Details about damage assessment in the civil cases are, unfortunately, not available. If the approach followed by Porter and Zona (1999) for the Ohio milk cases is of any indication, it appears that the overcharge was estimated by comparing bids in collusive and competitive school districts via reduced-form analysis.

\textsuperscript{13}See the discussion in Athey, Levin, and Seira (2011) for timber auctions.
The U.S. milk cartels were the subject of various papers in the empirical literature on bid-rigging. Pesendorfer (2000) examines the Florida and Texas school milk cartels and shows that the data (in particular market shares and incumbency rates) is consistent with a strong cartel in Florida (in the sense that side-payments were used between cartel members) and a weak cartel in Texas (no side-payments). This paper differs from the former in two dimensions: first, Pesendorfer focuses on the Dallas-Fort Worth (DFW) area whereas the dataset used in this paper includes, in addition, the Waco and San Antonio areas in which, according to the Department of Justice’s investigation, the cartel was not operating. These markets provide a set of competitive auctions which are useful to predict counterfactuals. Second, this paper focuses primarily on outsiders’ response to the cartel’s bidding behavior. Hewitt, McClave, and Sibley (1996) demonstrate that the high incumbency rates in Texas (the supplier of a given school district does not change from year to year in many cases) can only be explained by collusion. Lee (1999) finds evidence of complementary bidding and high incumbency premia in the DFW school milk market. Porter and Zona (1999) test for the presence of collusion in the Ohio school milk market by comparing defendants firms in Cincinnati to a control group of non-defendants and compute estimates of cartel damages. Lanzillotti (1996) provides a review of U.S. milk cartel cases and shows that several features of the bids in Kentucky are indicative of collusive behavior.

This paper is more broadly related to the empirical literature on bidding rings. A first strand in the literature focuses on the internal organization of bidding rings. Asker (2010) studies equilibrium bidding and side-payments in knock-out auctions held privately by the New York stamp cartel before the actual auction. Kwoka Jr (1997) analyzes bids and side-payments in knock-out auctions held by a real estate ring. A more recent strand of the literature, closer to this paper, studies the interaction of partial cartels and non-cartel firms. Clark, Coviello, and Shneyerov (2018) show that a cartel in Montreal’s asphalt market successfully deterred entry by outsiders (through threats and intimidation). The authors decompose the effects of bid coordination and entry deterrence on procurement costs and find that bid coordination dominates. Harrington, Hüscherlath, and Laitenberger (2016) analyzes how the German cement cartel controlled the expansion of non-cartel supply (from Eastern European countries) by sharing the collusive rents with German importers.

The empirical section of the paper relies on results from the literature on the structural estimation of auctions. In particular, the non-parametric estimation of the bidders' underlying cost distribution follows the methodology introduced by Guerre, Perrigne, and Vuong (2000). The empirical section makes use of some of the numerical methods developed in the computational literature on asymmetric auctions. Recent contributions in this literature are: Li and Riley (2007), Gayle and Richard (2008), and Fibich and Gavish (2011).

The paper is organized as follows. Section 2 describes the theoretical model. Section 3 presents the school milk market and the relevant factors affecting bidders’ cost structure. Section 4 describes the dataset. Section 5 examines the largest non-cartel firm’s bidding behavior through a reduced-form approach, and shows that all else equal, the largest non-cartel bidder bids less aggressively in school district where the cartel is operating. An assessment of cartel and umbrella damages is presented in Section 6 using a structural approach. Section 7 concludes.

2 A Theoretical Model

The section presents a model of first-price procurement auctions with asymmetric bidders and analyzes the effect of collusion, cartel size, and cartel mechanism on the non-cartel firm’s profits and bidding behavior.

The section builds on the theoretical literature on asymmetric first-price auctions. In particular, Maskin and Riley (2000) and Lebrun (1999, 2006) derive existence and uniqueness results, as well as comparative statics results for the equilibrium bid functions. This section is also related to the theoretical literature on bidding rings in first-price auctions. In particular, McAfee and McMillan (1992) characterize the optimal mechanism for strong and weak cartels, whereas Marshall and Marx (2007) compares first and second-price auction formats when a cartel is not all-inclusive and may not be able to control the bids of its members.

The case of a single non-cartel firm is considered for simplicity, and in anticipation of the empirical part. Throughout the theoretical section, I maintain the assumption that the existence of the cartel, members’ identities, and cartel mechanism are common knowledge to all players. Nevertheless, in the empirical application, non-cartel firms need not know of the cartel per se to bid optimally: a non-cartel firm needs to know only the equilibrium distribution of its lowest competing bid. The cartel having formed at least a decade prior

to the sample period, the assumption that non-cartel firms hold correct beliefs is reasonable and follows the prior literature (e.g., Pesendorfer (2000)).

Risk-neutral firms bid for a single contract in a first-price procurement setting. There is no reserve price. Denote by 1 the cartel bidder, and by 2 the non-cartel bidder. For \( i \in \{1, 2\} \), firm \( i \)'s cost \( c_i \) is drawn from a distribution \( F_i \) with support \([c, \bar{c}]\). \( F_i \) has a continuous density \( f_i \) strictly positive on \([c, \bar{c}]\). Cost are drawn independently across bidders and are private.

Denote by \( \beta_i \) bidder \( i \)'s equilibrium bidding strategy and by \( \phi_i = \beta_i^{-1} \) the corresponding inverse bid function. Note that if bidder \( j \) bids according to \( \phi_j \) and bidder \( i \) submits a bid \( b \), then the latter wins if and only if \( c_j > \phi_j(b) \). Thus, bidder \( i \)'s expected profit from bidding \( b \) is given by:

\[
\pi(b; c_i) = (b - c_i) \Pr(c_j > \phi_j(b)) = (b - c_i) \( 1 - F_j(\phi_j(b)) \). (1)
\]

The first-order condition with respect to \( b \) (at \( c_i = \phi_i(b) \)) is:

\[
\frac{f_j((\phi_j(b))}{1 - F_j((\phi_j(b)))} \frac{\phi_j'(b)}{b - c_i} = \frac{1}{b - c_i} \quad \text{with boundary condition } \phi_i(\bar{c}) = \bar{c}. \tag{2}
\]

Combining optimality conditions for the two bidders, the equilibrium bid functions solve the following system of differential equations:

\[
\begin{cases}
\frac{1}{b - \phi_1(b)} = \frac{f_2(\phi_2(b))}{1 - F_2(\phi_2(b))} \phi_2'(b) \\
\frac{1}{b - \phi_2(b)} = \frac{f_1(\phi_1(b))}{1 - F_1(\phi_1(b))} \phi_1'(b)
\end{cases} \tag{3}
\]

with right-boundary conditions: \( \phi_1(\bar{c}) = \phi_2(\bar{c}) = \bar{c} \). Maskin and Riley (2000) show that inverse bid functions must satisfy the additional condition that the minimum bid of all bidders is the same: \( \phi_1(b) = \phi_2(b) = c \) for some unknown \( b \).\(^{15}\)

Although analytical solutions of problem (3) are in general not available, one can derive

\(^{15}\)Court documents indicate cartel activities began as early as 1967. While non-cartel firms could hold incorrect beliefs, auction outcomes that are stable to learning and are generated by a Bayes-Nash equilibrium seem more plausible in this empirical setting.

\(^{16}\)The cartel bidder is the cartel member selected to bid in the auction on behalf of the cartel. Note that this setting does not rule out the possibility that other cartel members submit “non-serious” or “complementary” bids. If that is the case, the non-cartel firm knows that it is facing only one “serious” bid from the cartel.

\(^{17}\)I assume that the cartel mechanism does not dictate the cartel firm’s bid in the target auction. This assumption rules out mechanisms where side-payments distort bidding in the target auction, as in the case studied by Asker (2010). In his study comparing the Texas and the Florida milk cartels, Pesendorfer (2000) finds evidence consistent with the Texas cartel operating without side-payments. This assumption is further discussed in Appendix A.3.

\(^{18}\)Existence and uniqueness of an equilibrium in strictly increasing strategies was proved by Maskin and Riley (2000) and Lebrun (1996, 1999, 2006) among others.
properties of the equilibrium bid functions. The first property, proved by Maskin and Riley (2000) and Pesendorfer (2000), can be used to compare the cartel and non-cartel firms bid functions. Assume that the bidders’ cost distributions can be ordered according to hazard rate dominance, i.e.

\[
\frac{f_1(c)}{1 - F_1(c)} > \frac{f_2(c)}{1 - F_2(c)} \quad \text{for all } c
\]

This implies that, conditional on having a cost above \(c\), the cartel bidder is more likely to have a low cost than the non-cartel bidder.

**Proposition 1.** Under the hazard rate dominance assumption:

1. The cartel bidder bids less aggressively than the non-cartel bidder: \(\beta_1(c) > \beta_2(c)\) for all \(c \in (c, \infty)\)

2. Denoting by \(G_i\) bidder \(i\)’s equilibrium bid distribution (and \(g_i\) the corresponding density function), the cartel firm’s bid distribution dominates the non-cartel firms bid distribution (in the hazard rate sense): \(\frac{g_1(b)}{1 - G_1(b)} > \frac{g_2(b)}{1 - G_2(b)}\)

**Proof.** See Propositions 3.3 and 3.5 in Maskin and Riley (2000) or Krishna (2006).

Part 1 of the proposition implies that the non-cartel firm may win despite not having the lowest cost among the bidders. This results in an inefficient allocation. Part 2 of the proposition implies that the non-cartel firm’s equilibrium bid distribution first-order stochastically dominates the cartel firm’s equilibrium bid distribution. As noted by Pesendorfer (2000), the hazard rate dominance condition will be satisfied for instance when all firms (cartel and non-cartel firms) are ex-ante symmetric and the cartel mechanism is efficient, i.e when the cartel bidder has the lowest cost among cartel members. In this case, \(F_1(c) = 1 - (1 - F(c))^n\) where \(F\) is the ex-ante symmetric distribution of bidders, and \(n\) is the number of cartel members.

Studying the magnitude of umbrella damages requires the comparison of competitive equilibrium bid functions (i.e., bid functions when the firms do not collude) with the collusive bid functions derived above.

Denote by \(n\) the number of cartel members, or cartel size. For simplicity, assume that all bidders, whether or not in the cartel, are ex-ante symmetric. In the empirical analysis, I allow for the possibility that firms differ systematically in their costs of procurement based on bidder-specific characteristics (e.g., firm size, distance from the school district). All firms draw their cost from a distribution \(F\) with support \([c, \infty]\). \(F\) has a continuous density \(f\) strictly positive on \((c, \infty)\). Under these assumptions, the competitive auction in which firms
do not collude is a symmetric auction with \( n+1 \) bidders. This game has a unique equilibrium in strictly increasing strategies.

Denote by \( \beta \) the symmetric equilibrium bidding strategy, and by \( \phi = \beta^{-1} \) the corresponding equilibrium inverse bid function. In this case, if a bidder’s cost is \( c_i \), bidding \( b \) yields expected profits given by:

\[
\pi(b; c_i) = (b - c_i) \Pr(c_j > \phi(b), \forall j \neq i) = (b - c_i) (1 - F(\phi(b)))^n
\]  

(5)

The first-order condition with respect to \( b \) (at \( c_i = \phi(b) \)) is:

\[
\frac{nf(\phi(b))}{1 - F(\phi(b))} \phi'(b) = \frac{1}{b - c_i} \text{ with boundary condition } \phi(\bar{c}) = \bar{c}.
\]  

(6)

Let \( G(b) = F(\phi(b)) \) denote the distribution of a firm’s equilibrium bid. Let \( g(b) \) denote the corresponding density function. As bidders are symmetric, this distribution does not depend on \( i \). The first-order condition can be rewritten:

\[
c_i = b - \frac{1 - G(b)}{ng(b)}
\]  

(7)

Equation (7) expresses the individual private cost \( c_i \) as a function of the individual equilibrium bid \( b \), and the distribution of equilibrium bid \( G \). This mapping is used for the structural estimation of the cost distribution \( F \) from the observed distribution of bids \( G \) (see Guerre, Perrigne, and Vuong (2000) and step 2 in Section 6.2).

Comparison of the competitive benchmark to the collusive case reveals that the non-cartel firm’s interim profits (i.e., expected profits conditional on its cost) always increase when its competitors collude. Additionally, the magnitude of damages depends on the cartel mechanism: in particular, bounds on damages can be derived under an efficient and inefficient cartel for a rich class of cartel mechanisms. The detailed statements and proofs of these results along with comparative static results for the cartel size and mechanism are presented in Appendix A.

### 3 The School Milk Market

Most public school districts in Texas use procurement auctions to allocate contracts for the supply of school milk. Every year, between May and August, each district organizes a first-price procurement auction. Firms have one month to prepare their bids. Bidders have to sign a non-collusive affidavit, stating that they did not partake in any communication with other bidders regarding prices or participation and that they will not give or receive any...
side-payments. On the day of the letting, all submitted bids are opened, and the bidders’ identities and bids are publicly announced.

School district vary in contract characteristics, such as the estimated quantities of milk to supply, the number of delivery points, the contract period, the delivery times, etc. Dairy distributors have to deliver packaged milk to various customers (e.g., retail stores, government agencies, and schools). Retail stores are the primary revenue source for distributors. School milk contracts typically form 10% to 20% of a distributors’ revenue.

By nature, the school milk market is remarkably exposed to collusion. Firms compete only on prices as the terms of the contract (quantity and quality) are fixed and the product is homogeneous. There are many small contracts to be gained facilitating market division. Bids and bidders’ identities are publicly announced, which helps to detect price cuts by cartel members and increases the transparency of prices. Firms frequently interact as auctions are not held on the same day, which permits retaliation in case of cheating. The demand for milk is inelastic, so price-increases will yield higher profits and are unlikely to face any buyer resistance. Finally, the market is relatively concentrated, helping coordination.

Some of the aforementioned market features also enable potentially large umbrella damages. Indeed, these damages stem from non-cartel firms adjusting their bidding behavior to the supra-competitive levels sustained by the cartel (which also best-responds to non-cartel firms). This adjustment is feasible because all bids and bidders’ identities are publicly announced, which results in non-cartel firms learning the price level in the rigged districts and adapting to it.\footnote{Non-cartel firms need not know the identities of other competitors and the existence of the cartel to bid optimally: best-responding requires only knowledge of the distribution of the lowest competing bid.} The latter channel is reinforced by the high frequency of interactions, due to the large number of contracts every year.

Next, the cost structure of milk processors is described. In anticipation of the structural analysis, it is useful to decompose a milk processor’s cost for a specific contract into:

1. A component common to all firms bidding for a contract: this component depends on auction characteristics (e.g., quantity of milk to supply, the number of deliveries per week, etc.). Additionally, the common cost component depends on the price of raw milk, which is regulated by federal order, as well as processing, packaging and labor costs which, according to industry experts, are constant across firms in the market.\footnote{A federal milk order sets a uniform minimum price for raw milk in the area. This price is typically increasing in the distance from the Midwest. In this paper, the marketing area is the Texas Milk Marketing Order. Within the marketing area, prices differ by a fixed proportion from one zone to the other. The price of raw milk is around 7 cents per half-pint.}

2. An idiosyncratic component specific to the firm: this cost component depends on the
The idiosyncratic component also depends on the firm current capacity utilization. If the firm is near capacity, winning a new contract may signify employing an additional truck and driver, which largely increases the cost of fulfilling the contract. Finally, idiosyncratic costs include a firm’s efficiency in packaging, loading trucks, and managing the machinery.

A description of the conspiracy, as well as the main actors in the industry, is provided next. In 1992 and 1993, nine milk processors accused of collusion in the DFW market area settled with the State. The cartel included the leading suppliers with plants in the DFW market area: Borden, Foremost, Schepps, Cabell, Oak Farms, Metzger, Vandervoort, Gandy, and Preston. Indictments suggest that collusion began at least as early as 1967. The cartel went through the following structural changes: in 1983, Borden acquired Metzger; in 1985, Preston entered the school milk market and joined the conspiracy; in 1986, Schepps acquired Foremost; in 1990, Cabell acquired Oak Farms.

Pure Milk Co. is the largest non-cartel school milk supplier in the dataset. The firm’s main plant was located in Waco, TX (i.e., in a different federal order zone than the DFW cartel). The company was founded in the 1960s and was successful in establishing a strong local customer base in Central Texas by marketing higher quality dairy products. Its main raw milk supplier was Dairy Farmers of America, a national milk marketing cooperative. School contracts made up to 20% of the firm’s business.

Although Pure Milk bid primarily for contract near its plant in the Waco market area, it also participated in a non-negligible number of auctions for school districts in the DFW market area. According to the General Manager of the firm at that time, these were typically larger contracts justifying “going the extra mile.” In these occasions, Pure Milk bid against the cartel. The paper focuses on these contracts in which Pure Milk bid in districts where the cartel was operating. Pure Milk’s location further from the epicenter of the conspiracy may have played a non-negligible role in it not being part of the cartel. Most of its business was conducted in the Waco market area, whereas the cartel was only active in DFW.

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21The plant to school district distance is a proxy for the true transportation cost, which depends on how close the school district is to the firm’s distribution route. The distribution route depends on the firm’s current portfolio of clients (e.g., government agencies, military bases, retail stores, and schools).

22As in [Pesendorfer (2000)], collusion is thought of as an explicit or implicit scheme designed to limit competition and increase profits.

4 The Data

The paper studies school-milk contracts awarded annually between 1980 and 1992 in three market areas in Northeastern and Southern Texas: Dallas-Fort Worth (DFW), Waco and San Antonio. Contracts are awarded at the school district level. Figure 1 shows school districts in the dataset by market area. Each school district contains around four to five schools. Initially, the dataset contains information on 1,620 auctions, 4,444 bids.

The main dataset is the auction data. This dataset was collected by the Antitrust Division of the U.S. Department of Justice during its investigation of the Texas milk cartel. For each contract awarded, the following characteristics are observed: the county and school district awarding the contract, the identity of the bidding firms and corresponding bids for each milk category (whole white, whole chocolate, low-fat white, low-fat chocolate, skim milk), the quantities required per milk category, whether a cooler has to be provided, the number of meals served in the school district, the school district enrollment, the number of deliveries per week, the number of schools in the district, whether a bid was fixed or escalated, and the identity of the winner.

Three auxiliary datasets complement the auction data. Two are obtained from the U.S. Department of Agriculture’s Marketing Service. First, a dataset on prices of Class I fluid milk for the period of interest. This is the price of raw milk sold by milk cooperatives (such as Dairy Farmers of America) to milk processors and distribution firms. Prices differ from one region to the other within the Texas Milk Marketing Order. Second, a dataset giving the processing plant-locations of the firms bidding for the school milk contracts. Third, the longitudes and latitudes of school districts are added to the main auction data.

The reduced form and structural analysis are conducted on the data after the following preparation. Auctions with more than one winner are dropped (33 out of 1,620). Auctions with only one participant are dropped (514 out of 1,620). Prices are deflated using the CPI deflator into 1982 dollars. Finally, a distance variable is constructed for each observed bidder-auction pair: this variable measures the great-circle distance between the school district and the bidder’s closest plant. The variable is constructed using the latitude and longitude coordinates of firms’ plants and school districts. Auctions missing information on quantities

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24 Escalated bids are indexed to the price of raw milk, to ensure the milk supplier against potentially large fluctuations of the price of raw milk over the contract period.
25 The Southwest Market Area (Federal Milk Order 126).
26 Class I Milk is raw milk destined to be used as a beverage, in opposition to milk processed into yogurts, cheese, etc.
27 Without a public reserve price, modelling a one-bidder auction would require information on bidders’ perceived probability of their bid being rejected. Rejections are, however, rare in the data (8 bids are rejected in total). Note that a model with endogenous entry—in which potential bidders mix in their entry decisions—can generate one-bidder auctions. This possibility is discussed in details in Section 6.4.
or other relevant contract characteristics are dropped (39 out of 1,620). After this procedure, the dataset contains information on 1,034 auctions, 3,493 bids.

Table 1 shows that the majority of firms win between 20% and 30% of auctions in which they participate. Borden and Oak Farms are the largest firms by number of contracts won over this period. Pure Milk (the largest non-cartel firm) is in the second-tier of the distribution. It won 28% of the contracts on which it bid.

Table 2 gives summary statistics by market area. Over the period of interest, 735 contracts were awarded in Dallas-Fort Worth, 143 in San Antonio, and 179 in Waco. Only 15 school districts awarded contracts in San Antonio, against 30 for Waco. Indeed, school milk contracts in San Antonio are for larger quantities. The average winning bid (for a half-pint of whole white milk) is greater in DFW, followed by San Antonio, and lastly Waco. The average cost of a contract is the highest in San Antonio, reflecting again the larger size of contracts. The average contract cost in DFW is $85,115 against $18,779 in Waco. This cost difference reflects differences in contract sizes, raw milk prices, contract specifications across the two market areas but is also potentially related to inflated cartel prices in the DFW area.

5 Reduced Form Analysis

In this section, the bidding behavior of the largest non-cartel bidder (Pure Milk Co.) is examined. The analysis suggests that Pure Milk bid significantly less aggressively when facing cartel bidders and enables a preliminary assessment of the overcharge.

Pure Milk's processing plant is located near the city of Waco, in McLennan county. The firm bid primarily in McLennan and neighboring counties (see Figure 1). However, the firm also participated in auctions in counties located in the cartel area of activity (DFW). Auctions in which Pure Milk bid are divided by counties into two separate types:

- Collusive auctions: school districts located in counties contiguous or close to Dallas-Fort Worth in which the cartel presence was established by the Department of Justice. The counties are Comanche, Dallas, Erath, Hood, and Johnson. Such auctions form around 10% of Pure Milk’s bids.

- Competitive auctions: school districts located in counties outside the Dallas-Fort Worth cartel territory (either in the Waco or San Antonio market areas). The counties are Bell, Bosque, Comal, Coryell, Falls, Hill, Limestone, and McLennan.

This classification is based on the factual statement of the milk cartel prosecution. It is assumed that the cartel bid in the collusive counties, whereas all firms bid competitively in the competitive counties.
The logarithm of Pure Milk's bid for whole white milk is regressed on the variables listed in Table 3. All continuous variables are in logarithm. Quadratic terms for the number of meals and distance are included. In the first specification, no fixed effects are used. In the second specification, year fixed effects and dummy variables for “collusive” and “competitive” counties are used. In the third specification, year fixed effects and county dummy variables are used. Regression results are presented in Table 4.

Table 4 shows that bids move closely with the price of raw milk. As expected, escalated bids are lower than fixed bids (because they are indexed to the price of raw milk and therefore shield bidders against future fluctuations of their input price). Bids are increasing in the number of schools to supply within the district. Bids are decreasing in the number of bidders. In specifications (2) and (3), bids are increasing and convex in the number of meals served (equivalently in the quantity of milk to supply). In specifications (2) and (3), bids are concave in the distance between the processing plant and the school district. Distance increases the cost of fulfilling a contract, but with diminishing effects.

The results provide suggestive evidence that Pure Milk bid less aggressively when facing the cartel. Pure Milk bid on average 6.2% higher in the collusive auctions (facing the cartel) relative to the competitive auctions. Specification (3) breaks down the effect at the county level. As shown in Figure 2, the majority of coefficients are positive (and statistically significant) in collusive counties, whereas the coefficients are negative (and statistically significant) in counties with competitive auctions. All coefficients are measured with respect to the average bid in Bell county (competitive auction).

The set of collusive auctions may be selected and, therefore, not comparable to competitive auctions. Indeed, collusive auctions may differ from competitive auctions based (1) on contract characteristics or (2) on Pure Milk’s cost conditional on entering the auction (that is, if entry were endogenous). With respect to the first source of selection, the previous analysis controls for a rich set of observable auction characteristics, in particular the plant-school district distance and contract size. With respect to the second source of selection, I carefully consider the applicability of selective entry to this setting in Section 6.4 and find that the data does not support selection into auctions based on costs.

The reduced-form approach is limited if one is interested in the magnitude of umbrella

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28 The log-log specification allows the interpretation of coefficients as elasticities. Additionally, because bids are positive, errors are positively skewed. By logging the observed variable, errors are made more symmetric.

29 Distance is statistically significant only when year FE are included. This is due to the fact that, in the data, Pure Milk’s bids (in 1982 $) follow a negative time trend whereas the average plant-school district distance increases over time.

30 If Pure Milk’s probability of entry is higher in collusive auctions, its expected cost (and bid) conditional on entry could be higher in collusive auctions.
damages relative to cartel damages, or in assessing the size of inefficiencies introduced by the cartel agreement. Therefore, more structure is imposed on the data. This structure is derived from the theoretical model of Section 2. The next section shows how the structural model and the bid data can be used to simulate counterfactual bids and assess cartel and umbrella damages.

6 Structural Analysis

In this section, bidders’ underlying cost distributions are estimated from the bid data. These estimates are used to simulate a set of counterfactual auctions and assess, first, the size of damages caused to school districts by the outsider firm when facing the cartel, and second, inefficiencies in contract allocation caused by asymmetries between cartel and non-cartel bidders. Before presenting the details of the estimation approach and counterfactual results, the next subsection discusses practical issues in selecting the sample of auctions used for estimation.

6.1 Practical Considerations

Although the dataset used is rich in many dimensions, not all auctions at hand can be used to estimate firms’ underlying cost distribution. The structural model developed in Section 2 requires (at least) two firms bidding non-cooperatively in each auction. Based on the list of firms prosecuted by the Department of Justice, all firms with plants located in the DFW area were colluding for contracts in that area. As a consequence, the majority of bids for auctions in DFW (in which all participants were cartel members) were either (1) a winning bid submitted by the cartel bidder or (2) cooperative “phony” (or “complementary”) bids submitted by other cartel bidders. The structural model cannot be used to infer underlying costs from these bids, as they do not necessarily map to a firm’s true cost.\footnote{In auctions where all bidders are cartel members, the cartel bidder likely chooses the highest mark-up possible without raising suspicion, inducing defection, or entry by additional firms. Without knowing the cartel’s beliefs about these threats, one cannot confidently map the cartel winning bid to its underlying cost.}

A natural way to circumvent the previous issue would be to restrict the sample to the subset of collusive auctions in DFW in which the outsider firm bid against the cartel. Unfortunately, two reasons make this alternative unappealing.\footnote{\cite{Pesendorfer} finds evidence that the Texas cartel was quasi-efficient even though side-payments were not used, by comparing it to the Florida cartel which was efficient.}

First, the cartel mechanism (i.e., how the cartel bidder is selected) is a priori unknown.\footnote{Even if the cost distributions for the outsider and cartel bidder are recovered, the cost}
distribution of other cartel members cannot be recovered without knowledge of the cartel mechanism. As a consequence, the counterfactual competitive auction, in which all cartel members bid non-cooperatively, cannot be simulated.\footnote{One could assume a mechanism for the selection of the cartel bidder, and estimate underlying costs from observed bids. However, if the mechanism is misspecified, this approach will lead to biased estimates of underlying costs.} Second, the small sample size of this set of auctions (in which the outsider bids against the cartel) renders any non-parametric estimation of the distribution of costs unavailable.\footnote{The outsider firm bid against the cartel in 33 auctions.}

In the preferred approach, auctions from the two other market areas in the dataset (Waco and San Antonio) are used. As no firm was prosecuted in counties around Waco and San Antonio, I assume that firms were bidding competitively in these counties (denoted hereafter “competitive” auctions). The set of competitive “normalized” bids (i.e., controlling for differences in contract characteristics) can be used to recover the distribution of “normalized” costs. Using the estimated distribution of costs, I simulate counterfactual bids by solving the asymmetric auctions in which the outsider firm bids against the cartel, for two polar choices of the cartel mechanism (efficient cartel or inefficient cartel). Appendix \ref{app:A.3} shows that these two choices provide bounds on damages within a rich class of cartel mechanisms.

The main identification assumption is that, conditional on contract and bidder specific characteristics (contract observables, plant to school district distance, etc.), the idiosyncratic cost distribution of a given firm is identical in competitive and collusive markets: a firm’s cost distribution does not depend on whether it is colluding or bidding competitively. Under this assumption, bids submitted in competitive auctions can be used to infer firms’ underlying cost distributions and simulate outcomes in rigged school districts.

Advantages of this approach are twofold. First, estimation of the cost distribution does not hinge on the correct specification of the cartel mechanism. Indeed, costs are obtained from competitive auctions in which all firms bid non-cooperatively. Second, the cost distribution is estimated non-parametrically, as the sample size of competitive auctions allows it.\footnote{This approach relies on data from competitive auctions being available. Such data would typically be available at the time when the distinction between cartel and umbrella damages is most relevant, i.e., when assessing liability for damages. By this stage in the investigation, the antitrust agency would have used its expertise and potential leniency offers to identify the extent of the cartel’s operations.}

A drawback of this approach is that it does not recover the specific cost corresponding to observed bids for collusive auctions in which the outsider participates. Instead, it computes upper and lower bounds (assuming an inefficient and efficient cartel mechanisms respectively) on damages using (1) the (normalized) cost distribution estimated from the set of competitive auctions and (2) auction observed characteristics drawn from collusive auctions in which the
outsider participates. Moreover, this approach assumes that the distribution of firm-specific (normalized) costs is identical in competitive and collusive districts.

6.2 Estimation Approach

The following assumptions are imposed in the baseline estimation approach. Denote by $\mathbf{x}_d$ a vector reporting auction $d$’s observed characteristics (such as the quantity to be supplied, whether bids can be escalated, number of deliveries per week, number of schools in the district, date, etc.).

**Assumption 1 (Multiplicative Separability).** *Bidder $i$’s cost in auction $d$, denoted $c_{id}$, can be written:*

$$c_{id} = \tilde{c}_{id} \Gamma(\mathbf{x}_d)$$

*for some function $\Gamma(\cdot)$. $\tilde{c}_{id}$ represents bidder $i$’s idiosyncratic (“normalized”) cost in auction $d$, which is independent from $\mathbf{x}_d$.*

**Assumption 2 (Symmetric IPV).** *Idiosyncratic costs $\tilde{c}_{id}$ are symmetric (identically distributed according to the distribution $F$), independent, and private across firms, school districts, and time.*

Section 6.4 relaxes the symmetry and multiplicative separability assumptions. The steps followed in the estimation are described below.

- **Step 1: Observed auction heterogeneity**
  The estimation procedure assumes that the data available is from auctions of ex-ante identical contracts. This assumption is not valid for school milk contracts. Indeed, contracts differ in various dimensions, which are public information and observed by the bidders before submitting their bids. This public information will enter not only a bidder’s private cost of realizing the contract but also its belief about other bidder’s costs.

There are several possible ways to account for observed auction heterogeneity. In the baseline estimation, I follow the approach of Balat et al. (2016) and explores alternatives in Section 6.4. The latter paper shows that separability (Assumption 1) of idiosyncratic costs and auction characteristics carries out to bids.

**Lemma 1.** Assume the multiplicative separable structure (Assumption 1), the equilibrium bid function has the multiplicative separable form:

$$\beta(c_{id}) = \beta(\tilde{c}_{id}) \Gamma(\mathbf{x}_d)$$

---

36See Haile, Hong, and Shum (2003) for an early version of the paper.
This result can be used to account for observed auction heterogeneity and homogenize the bids. Assume the following parametric specification: \( \Gamma(x_d) = \exp(x_d'\alpha) \). The first-stage regression is:

\[
\ln b_{id} = x_d'\alpha + \eta_t + \kappa_i + \sum_{n=2}^{6} 1(n_d = n)\delta_n + \sigma_{id}
\]  

where \( b_{id} \) denotes the bid of bidder \( i \) for contract \( d \), \( \eta_t \) is a time-specific dummy, \( \kappa_i \) is a bidder-specific dummy, \( n_d \) is the number of bidders participating in contract \( d \), and \( \sigma_{id} \) is the error term. \( x_d \) include variables for the price of raw milk, the number of meals served (and its square), whether bids are escalated, whether a cooler has to be provided, the number of deliveries per week, and the number of schools in the school district. All continuous variables are in logarithm. Time dummies capture seasonality: for instance, common packaging, processing, and labor costs, that might change over time but are common to all bidders.

The first-stage regression is run on the sample of competitive auctions from the Waco and San Antonio market areas, in which all firms bid non-cooperatively.

Normalized bids are constructed from the results of regression (8), as \( \ln \tilde{b}_{id} = \ln b_{id} - x_d'\hat{\alpha} - \hat{\eta}_t \).

As equilibrium bid functions depend on the number of bidders participating in the auction, the rest of the estimation (in particular, when inverting the first-order condition) is conducted on auctions fixing the number of participants to three bidders. Section 6.4 discusses results for auctions with more than three bidders. In particular, Section 6.4 allows the number of cartel bidders (and therefore the distribution of the efficiently selected cartel bidder) to vary across auctions.

- **Step 2: Estimation of the underlying cost distribution**

Following the procedure presented in Guerre, Perrigne, and Vuong (2000), the underlying distribution of costs can be estimated using the distribution of normalized bids, obtained in the previous step, by inverting the first-order condition.

Cumulative distributions are estimated using the empirical distribution function and the densities are estimated following the boundary-corrected approach of Hickman and Hubbard (2015).\(^{37}\) This step also recovers the equilibrium bid function in competitive auctions under Assumption 2.

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\(^{37}\)This approach is based on the contributions of Zhang, Karunamuni, and Jones (1999) and Karunamuni and Zhang (2008). Pinkse and Schurter (2019) discusses issues with the assumptions on the necessary number of derivatives at the boundary and the choice of the first bandwidth parameter.
Step 3: Derivation of the equilibrium bid functions in collusive auctions

As shown in Section 2, the cartel bidder and non-cartel firm equilibrium inverse bid functions, denoted $\phi_1$ and $\phi_2$ respectively, are the solutions of:

$$
\begin{align*}
\frac{1}{b - \phi_2(b)} &= \frac{f_1(\phi_1(b))\phi_1'(b)}{1-F_1(\phi_1(b))} \\
\frac{1}{b - \phi_1(b)} &= \frac{f_2(\phi_2(b))\phi_2'(b)}{1-F_2(\phi_2(b))}
\end{align*}
$$

(9)

where $F_i$ (resp. $f_i$) is the cumulative distribution (resp. density function) of costs of the cartel bidder ($i = 1$) and the outsider firm ($i = 2$). Along with the boundary conditions $\phi_1(b) = \phi_2(b) = c$ (lower bound of support of bids) and $\phi_1(\bar{c}) = \phi_2(\bar{c}) = \bar{c}$, the system (9) forms a nonlinear boundary value problem (BVP), in which the location of the left-boundary $b$ is unknown. In addition to this latter “non-standard” feature, the numerical resolution of the BVP is further complicated by the fact that the mapping $M$ in $(\phi_1'(b), \phi_2'(b)) = M(b, \phi_1(b), \phi_2(b))$ is not Lipschitz-continuous in $(\phi_1(b), \phi_2(b))$ at the right boundary $\bar{c}$.

The BVP defined in equation (9) cannot be solved analytically. However, several numerical solutions have been proposed in the literature: the backward-shooting method was first used to solve this problem by Marshall et al. (1994) and used and refined in various subsequent papers. Although this method is currently the standard for computing equilibrium bids in asymmetric auctions, it suffers from large instability at the right boundary (see Fibich and Gavish (2011) for a detailed analysis). As a consequence, an alternative numerical method, proposed in the latter paper, is preferred. Their idea is to recast the BVP as a system of differential equations in $\phi_1$ and $b$ (instead of $\phi_2$) as functions of $\phi_2$. This allows to transform the BVP with unknown left boundary into a BVP with known boundaries, and apply standard numerical techniques such as fixed-point iteration on a grid. The details of their numerical method, adapted to this setting, are presented in Appendix B.

As the cartel mechanism is unknown (regarding how the cartel bidder is selected for a given target auction), two extreme cases for the cartel internal mechanism are considered. These two scenarios provide upper and lower bounds on damages within a rich class of possible cartel mechanisms. Appendix A.3 provides a formal proof of this result. The intuition for the result is as follows. As the cartel mechanism becomes more “efficient” (in a sense that I make precise in Appendix A.3), the non-cartel firm faces a

\[\text{Gayle and Richard (2008) use local Taylor series expansions of the solution and the distribution and Li and Riley (2007) use an adaptive step size for the numerical backward integration to allow better control of the error.}\]
stronger cartel bidder and bids more aggressively. Additionally, the cartel bidder’s bid distribution shifts to the left (i.e., becomes stochastically dominated). The combined effect is that procurement costs and damages decrease overall and conditional on the non-cartel firm winning against the cartel.

Under an efficient cartel mechanism, the cartel selects its lowest cost member to bid on behalf of the cartel at the auction. Let $F$ denote the true distribution of costs for each bidder. Then the cost distributions of the non-cartel firm and the cartel bidder are:

$$F_1(c) = 1 - (1 - F(c))^n \quad F_2(c) = F(c) \quad \forall c \in [\underline{c}, \overline{c}]$$

where, in the current application using three-bidder auctions, $n$ equals two.

Under an inefficient cartel mechanism, the cartel bidder is selected randomly among cartel members. Then the cost distributions of the non-cartel firm and the cartel bidder are:

$$F_2(c) = F_1(c) = F(c) \quad \forall c \in [\underline{c}, \overline{c}]$$

Estimators of $(F_1, F_2)$ are constructed from the non-parametric estimators of $F$ obtained in Step 2. Given these estimates, the BVP is solved using a fixed point iteration algorithm. The result of the numerical method are estimators of the true asymmetric equilibrium inverse bid functions: $\hat{\phi}_1$ and $\hat{\phi}_2$.

The equilibrium bid functions are numerically obtained assuming common knowledge of the cartel mechanism and cartel members (as evident in the system of differential equations). It is worth noting that, in practice, Pure Milk need not know of the cartel’s existence or members’ identities per se to bid optimally: only knowledge of the equilibrium distribution of its lowest competing bid is required. Given that the cartel formed at least a decade prior to the sample period, it is reasonable to assume that auction outcomes are generated by a Bayes-Nash equilibrium that is stable to learning and in which the non-cartel firm holds correct beliefs (Pesendorfer (2000) makes a similar assumption).

- **Step 4: Damage Assessment**

  Given estimates of the distribution of interest, I simulate $L$ three-bidder auctions by

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39 Whereas the definition of an efficient cartel mechanism is straightforward, it is less evident for an inefficient cartel mechanism. In the inefficient case, the cartel bidder could be randomly selected as is assumed in this paper. However, one could think of other inefficient mechanisms such as the one in which the cartel bidder is the least cost-efficient member. Appendix A.3 discusses reasons why the former definition of an inefficient cartel is more plausible than the latter in this particular application.

40 In the inefficient cartel case, the auction is symmetric and a closed-form solution for the equilibrium bid function is available.
(1) drawing costs from the estimated cost distribution, (2) constructing the corresponding bids in the competitive auction and collusive auctions under the two cartel mechanisms, (3) reincorporating observed auction heterogeneity by drawing from the empirical distribution of auction characteristics in cases where the outsider firm bid against the cartel.

Estimates of damages are constructed by comparing the set of simulated competitive winning bids and corresponding counterfactual set of collusive winning bids where the non-cartel firm faces the cartel. Expected damages are obtained by averaging damages over the \( L \) simulated auctions (in practice, \( L = 5,000 \)). Estimates of efficiency losses are constructed by comparing the winner’s cost in the competitive and corresponding collusive simulated auctions and averaging the differences across the \( L \) simulations.

### 6.3 Assessment of Damages and Inefficiencies

This section presents the results of the counterfactual analysis. Figure 3 shows the estimated bid functions in the case of three-bidder auctions. Each panel shows the bid function in the competitive auction, along with the counterfactual outsider and cartel’s bid functions in the case of an efficient cartel mechanism (left panel), and an inefficient cartel mechanism (right panel). In both cases, the collusive bidding functions lie above the competitive bidding function as both the cartel bidder and the outsider bid less aggressively. The mark-up above the competitive bid increases the smaller is the bidder’s cost. The non-cartel firm’s mark-up when the cartel is inefficient is larger than when the cartel is efficient (this is consistent with the theoretical results of Appendix A.2).

Estimates of damages to the auctioneer (school district) correspond to the difference between the winning bid of the collusive auction in which the outsider firm bids against the cartel and the winning bid of the competitive auction in which all firms bid non-cooperatively. Cartel damages are the typical damages antitrust authorities aim to assess, that is, for auctions won by the cartel. Umbrella damages, on the other hand, are computed for auctions where the outsider wins against the cartel.

Table 5 presents damage estimates obtained from the structural analysis. The mean damages per contract in the whole white milk category are between $399 and $862. If the same overcharge were applied to all milk categories, damages per contract would be between

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41 Normalized bid functions \( \hat{\beta} : \hat{c}_{id} \rightarrow \beta(\hat{c}_{id}) \) are represented.
42 Umbrella damages (resp. cartel damages) are computed conditional on the outsider (resp. the cartel) winning the collusive auction.
43 I focus on the whole white category because it is the most frequent category. Other milk categories, such as whole chocolate, are only part of a subset of contracts.
For auctions won by the outsider, the collusive winning bid is 1.6% to 6.4% above the competitive winning bid, depending on the cartel mechanism. As a fraction of the winning competitive mark-up, these damages are between 23% and 62%. For auctions won by the cartel, damages are between 4.9% and 6.4% of the competitive winning bid, or between 55% and 62% of the competitive winning mark-up.

Inefficiencies introduced by the (efficient) cartel mechanism are measured by the difference in the winner’s cost in the competitive and collusive auctions. These two costs differ when the outsider firm wins the collusive auction, while the cartel would have won the competitive auction. Inefficiencies amount to an increase of 5.9% of the winner’s cost, equaling $2,337 per contract.44

Umbrella damages form a non-negligible fraction of cartel damages. Per contract, umbrella damages (conditional on the outsider winning) are estimated to be at least 35% of cartel damages (conditional on the cartel winning). This lower bound is obtained with an efficient cartel. If the cartel is inefficient, umbrella damages are as large as cartel damages. The estimates found for umbrella damages as a fraction of the competitive winning bid, between 1.6% and 6.4%, are of similar magnitude as the overcharge of 6.2% estimated in the reduced form section, as well the overcharge found by Porter and Zona (1999) for the Ohio school milk market.

6.4 Modeling Alternatives and Extensions

The econometric model estimated and the results reported above rely on maintained assumptions 1 and 2. In this section, I relax these assumptions and incorporate endogenous entry into the baseline model.

Asymmetries between bidders and observed auction heterogeneity. A bidder’s idiosyncratic cost of serving a given school district is affected by several bidder-specific factors: the distance between its closest processing plant and the school district, the bidder’s capacity (e.g., trucks and drivers), and the bidder’s efficiency in packaging, loading trucks, managing the machinery. Idiosyncratic costs are private to each bidder, as they do not affect its competitors’ costs. Moreover, these factors are independent across bidders.

44This calculation is valid if, for a given total bid (inclusive of quantities), bids per half-pint by category are determined by applying the same mark-up over costs per half-pint for all milk categories. This pricing rule seems more reasonable than alternatives, for instance, a mark-up imposed on a subset of categories while the remaining categories are priced at cost.

45If the cartel is inefficient, misallocation can arise. However, this is due to the random selection of the cartel bidder rather than asymmetries between bidders. If a cartel member would have won the competitive auction and is not selected as the cartel bidder, there will be a loss of efficiency. This type of misallocation is left aside as it is not stemming from asymmetries.
Several bidder-specific factors are, however, public information and create ex-ante asymmetries across bidders. For instance, bidders (plants) located near a given school district are likely to have lower transportation costs and, hence, lower costs of fulfilling the contract; moreover, firm size may affect efficiency. Table 1 shows large differences in the number of auctions bidders participate in and win. Some firms such as Borden and Oak Farms are national with a large distribution network, whereas others, such as Pure Milk, only operate in parts of Texas.

In this section, I allow for the possibility that firms differ systematically in their costs of procurement based on their plant-to-school district distance and capacity. A firm’s capacity is defined as the maximum total dollar value won by the firm in a year (including both rigged and competitive school districts). For each auction, bidders are classified into three types: small firms located near the school district, large firms located near the school district, and large firms located far from the school district. I use different thresholds for distance (30 to 80 miles by 10-mile increments) and firm size (25th, 50th, and 75th percentiles of the distribution of capacities by year).

Additionally, observed auction heterogeneity is controlled for using an alternative approach (Step 1). I assume that the log of bidder i’s bid in auction d, denoted \( \ln(b_{id}) \), is normal with mean and variance given by

\[
\begin{align*}
E[\ln(b_{id})|x_d, n_d] &= [x_d, n_d]^{'} \alpha_{k(i,d)} \\
\text{Var}[\ln(b_{id})|x_d, n_d] &= \exp(z_d^{'}\delta_{k(i,d)})^2
\end{align*}
\]  

(10) (11)

where \( z_d \) is a subset of contract characteristics entering \( x_d \), \( k(i, d) \) is bidder i’s type in auction \( d \), \( n_d = (n_1, n_2, n_3) \) are the number of bidders by type.

I specify the mean of log bids as a linear function of all contract characteristics entering \( x_d \) and include year dummies and dummies for the number of bidders by type. I allow the effects of most of these covariates to differ by bidder type. The variance of log bids depends on variables \( z_d \) capturing contract size (number of meals and number of school in the district) and the bidder’s type. The coefficients \( \alpha_{k(i,d)} \) and \( \delta_{k(i,d)} \) are estimated by maximum likelihood using the sample of competitive auctions (San Antonio and Waco market areas).

For each draw of \((x_d, n_d)\), I follow the rest of the estimation approach of Section 6.2.

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46 There are no small firms located far from the school district in the data.
47 The cost estimates obtained using the homogenization approach of Balat et al. (2016) may not be robust to small departures from the maintained separability assumption.
48 This approach follows the literature relying on parametric assumptions in the estimation of auction models. See, for instance, Athey, Levin, and Seira (2011) and Krasnokutskaya and Seim (2011).
allowing cost distributions to differ by bidder type. To simulate the counterfactual collusive auctions, auction characteristics $x_d$ are drawn from the set of auctions in which Pure Milk bid against the cartel. Additionally, I assume that there is one outsider (the bidder with the weakest type) whereas all remaining firms collude efficiently.

Table 6 (Panel A) presents damage estimates (as a fraction of the competitive winning bid) averaged over auctions ($x_d$) in which Pure Milk bid against the cartel, for different values of $n_d$. The thresholds to define bidder types are 70 miles and the median capacity by year. I find damages of the same magnitude as in the baseline estimation approach: for instance, if $n_1 = 1$ and $n_2 = 2$, cartel damages are 4.9% and umbrella damages are 3.6%. As expected, damages increase with the number of bidders per auction. The predictions are qualitatively similar if I use alternative thresholds for distance and firm size.

Unobserved heterogeneity. The estimation approach abstracts from unobserved auction heterogeneity. The latter would be relevant if bidders were to observe auction characteristics that are unobserved by the econometrician. According to industry experts, there do not seem to be important common factors relevant to firms’ costs aside from the ones controlled for in the first estimation step, i.e., input prices, quantities, and auction specifications (escalated bids, coolers, number of deliveries, etc.).

Endogenous entry. Another issue concerns endogenous entry by non-cartel firms. High collusive prices in the DFW market area may have attracted Pure Milk to enter and bid against the cartel. Whereas, under competitive conditions, the firm would not have entered and bid on such contracts, in particular, because they are located further from its main Waco plant. The counterfactual competitive winning bid should, therefore, account for the possibility that Pure Milk (or some other potential bidders) may not have entered absent collusion. Ignoring this endogenous entry channel can result in inflated estimates of cartel and umbrella damages.

To account for endogenous entry, I augment the previous bidding model (with asymmetric bidders) with an initial entry stage, following other applications in the literature, e.g., Athey, Levin, and Seira (2011) and Krasnokutskaya and Seim (2011). At the entry stage, potential bidder $i$ decides whether to participate in the auction given knowledge of its private entry cost, the distribution of type-specific contract and entry costs, and the number of potential

---

49 The outsider is always the weakest type among firms participating. For instance, if $n_1 = 1$ and $n_2 = 2$, the outsider firm is of type 1 whereas colluding firms are of type 2.

50 Even if entry costs were small (relative to contract value) according to industry experts, capacity constraints would force firms to submit bids only on a subset of contracts. These contracts would be the most profitable for the firm ex-ante, e.g., located closer to the firm’s plants.

51 Defendants (in this case, cartel members) would argue that non-cartel firms should be excluded when simulating competitive prices, as this would yield higher competitive winning bids and, therefore, lower damages.
bidders by type. Entry decisions are made simultaneously and prior to knowing contract costs. Firms with expected profits above their entry cost enter the auction. Entrant \( i \) learns its private contract cost and the number of other entrants by type, and submits its bid.

The assumption that entrants know the number of competitors is consistent with the fact that auctions are held on-site and follows the previous literature on the milk cartel cases (e.g., Porter and Zona (1999) and Pesendorfer (2000)). The assumption that a potential bidder does not observe her contract cost realization at the time of her entry decision but learns it through bid preparation is imposed for two reasons. First, bids do not depend on the number of potential bidders, conditional on auction characteristics and the number of entrants \( (x_d, n_d) \) (the coefficients on the number of potential bidders by type when included in Equation (10) are not statistically significant). If bidders were to observe a signal about their contract costs prior to entry (i.e., entry is selective), then one would expect an increase in the number of potential bidders to be negatively associated with bids–conditional on \( (x_d, n_d) \). Second, this choice simplifies the empirical implementation of the model: the lack of selection on costs allows me to estimate the untruncated contract cost distribution from competitive auctions and apply it to collusive auctions.

Let \( N = (N_1, N_2, N_3) \) denote the number of potential bidders by type, and \( n = (n_1, n_2, n_3) \) the number of entrants for a given contract. Given an equilibrium of the bidding stage, denote by \( \pi_k(c_i; n) \) the equilibrium interim payoff of a bidder with cost \( c_i \) and type \( k \in \{1, 2, 3\} \) under entry combination \( n = (n_1, n_2, n_3) \) (abusing notation, \( \pi_k \) accounts for the fact that the actual number of type-\( k \) competitors is \( n_k - 1 \)). Under type-symmetric entry probabilities \( p = (p_1, p_2, p_3) \), expected profits at the entry stage are given by

\[
\pi_k(p) = \sum_n \int \pi_k(c_i; n) dF_{C,k}(c_i) \Pr(n_k - 1, n_k | N) \tag{12}
\]

where \( F_{C,k} \) is the contract cost distribution of type \( k \) bidders and \( \Pr(n_k - 1, n_{-k} | N) \) is the probability that there are \( n_k - 1 \) entrants of type \( k \) and \( n_{-k} \) entrants of the other types, given \( N = (N_1, N_2, N_3) \) potential entrants. The number of entrants follows a binomial distribution, for example if \( k = 1 \),

\[
\Pr(n_1 - 1, n_2, n_3 | N) = C_{N_1-1}^{n_1-1} p_1^{n_1-1} (1-p_1)^{N_1-n_1} C_{N_2}^{n_2} (1-p_2)^{N_2-n_2} C_{N_3}^{n_3} (1-p_3)^{N_3-n_3} \tag{13}
\]

---

52 Entry models with heterogeneous entry costs (such as in Moreno and Wooders (2011)) better fit the data in this application compared to alternatives such as the model of Levin and Smith (1994). Indeed, all bidder types are observed to mix in their entry decisions, whereas a model with common entry costs would predict that the strongest type always enter.
where \( C^n_N \) denotes the binomial coefficient of choosing \( n \) firms out of \( N \) potential bidders. Potential bidders enter if their entry cost is below their expected profits from entering \( \pi_k(p) \). Each potential bidder \( i \) of type \( k \) draws an entry cost \( K_i \) from the type-specific distribution \( G_k \). In equilibrium, bidders’ belief about entry probabilities are correct and these probabilities solve the following system of equations

\[
p_k = G_k(\pi_k(p)) \quad \text{for} \ k \in \{1, 2, 3\}
\]  

Moreno and Wooders (2011) and Krasnokutskaya and Seim (2011) provide equilibrium existence results for entry models with heterogeneous costs. Due to equilibrium multiplicity in the entry stage, as is standard in the literature, I assume that the auction data is generated by the same entry equilibrium.

Following the literature on entry, I define potential bidders for a given contract as the observed entrants plus any firm that submitted a bid in any auction for a school district within 60 miles of the school district in question in that given year. I find that around 97% of observed bidders also bid in another auction within 60 miles in the same year indicating that this definition captures most firms who would have participated with some positive probability.

Entry probabilities are estimated from the set of competitive auctions (Waco and San Antonio market area) using the following logit model

\[
p_k(i,d) = \frac{\exp([x_{d}, N_d]'\gamma_k(i,d))}{1 + \exp([x_{d}, N_d]'\gamma_k(i,d))}
\]  

where \( x_d \) is the same set of contract characteristics used in estimating the bid distribution in Equation (10), \( k(i, d) \) is potential bidder \( i \)’s type in auction \( d \), \( N_d = (N_1,d, N_2,d, N_3,d) \) are the number of potential bidders in auction \( d \) by type. The median number of potential entrants is four, whereas the median number of entrants is three. The median and mean entry rates are close to 65%.

Damages are computed conditional on Pure Milk entering and bidding against the cartel in the collusive auction. For the collusive auction, I assume that in addition to the cartel bidder, all other potential cartel bidders enter and submit complementary bids (Pesendorfer (2000) makes a similar assumption). For the competitive auction, I simulate counterfactual entry and bidding decisions using estimated equilibrium entry probabilities and (asymmetric) equilibrium bid functions given the number of entrants.

Given a combination of observed auction characteristics \( x_d \) from auctions where Pure

---

53 Roberts and Sweeting (2016) use a similar approach to define potential entrants.

54 Estimation of entry costs is not necessary as as they do not enter the counterfactual analysis.
Milk bid against the cartel, an auction is simulated using the following procedure

1. Choose the number of potential entrants by type \( N_d = (N_{1,d}, N_{2,d}, N_{3,d}) \). To ease computation, the analysis is restricted to auctions with at most two types.

2. Simulate outcomes of the collusive auction by drawing from bidders’ cost distributions and applying equilibrium bid functions (corresponding to the number/type of cartel bidders and non-cartel firm) to the costs of Pure Milk and the cartel bidder. The cartel mechanism is assumed efficient and outcomes are conditional on Pure Milk bidding against the cartel. Denote by \( b_{w}^{\text{col}} \) the winning bid in the collusive auction.

3. Simulate outcomes of the competitive auction by, first, drawing the vector of entrants \( n_d = (n_{1,d}, n_{2,d}, n_{3,d}) \) (given entry probabilities \( p_k(x_d, N_d) \) for \( k \in \{1, 2, 3\} \)), and second, applying equilibrium bid functions (corresponding to the number/type of active entrants) to the costs of entrants using the same cost draws as in the previous step. Denote by \( b_{w}^{\text{com}} \) the winning bid in the competitive auction.

4. Compute damages as a fraction of the competitive winning bid as

\[
\frac{b_{w}^{\text{col}} - b_{w}^{\text{com}}}{b_{w}^{\text{com}}}
\]

I distinguish between instances where the non-cartel firm wins the collusive auction (umbrella damages) and instances where the cartel wins the collusive auction (cartel damages).

For each choice of \( x_d \) and \( N_d \), \( L \) auctions are simulated using the aforementioned procedure and damages ((\( b_{w}^{\text{col}} - b_{w}^{\text{com}} \)/\( b_{w}^{\text{com}} \)) are averaged across the \( L \) simulated auctions (in practice, \( L = 5,000 \)). Finally, for each \( N_d \), I average damages over the combinations of observed auction characteristics \( x_d \) from auctions where Pure Milk bid against the cartel. The results are presented in Table 6 (Panel B).

The magnitudes of cartel and umbrella damages decrease when entry is endogenous, by an average of 55%. While the collusive winning bid is not affected by entry (since, by assumption, damages are computed conditional on Pure Milk entering and bidding against the cartel), the competitive winning bid is in expectation higher because some potential bidders (Pure Milk or cartel members) do not participate in the competitive auction. However,

55If there is only one realized entrant, I follow Krasnokutskaya and Seim (2011) and assume that the auctioneer submits a random reserve price drawn from the equilibrium bid distribution of the weakest potential-entrant type. Therefore, the winning bid is determined as in a two-bidder auction, which is known by the entrant.
conditional on the non-cartel firm winning the collusive auction, umbrella damages are still significant compared to cartel damages. Interestingly, in two instances \((N_2, N_3) = (3, 0)\) and \((N_2, N_3) = (1, 2)\), umbrella damages are negative, that is, the competitive winning bid is higher than the collusive winning bid (submitted by the non-cartel firm). This occurs because competitive bidding does not outweigh the combination of high costs for cartel members (since the non-cartel firm wins the collusive auction) and the drop in the number of entrants into the competitive auction.

Taken together, these results indicate that incorporating endogenous entry into damage assessments is important especially in industries where firms compete for a large number of contracts. In the case of the school milk industry, I find that estimated entry rates are relatively high (around 65% per auction), therefore, umbrella damages are still important even when non-cartel firms’ entry decision absent collusion is accounted for. Nonetheless, in other industries characterized by higher entry costs relative to contract value, the level of counterfactual competitive bids may be more significantly affected by entry.

7 Conclusion

This paper examines how non-cartel firms’ bidding behavior can be affected by the existence of a cartel in a first-price procurement setting. In the case of the Texas school milk cartel, the analysis shows that the largest non-cartel firm bid significantly higher when facing the cartel than under competitive conditions. The structural model reveals that, conditional on the non-cartel firm winning against the cartel, damages to the auctioneer (in the form of inflated winning bids) are a non-negligible fraction of the damages caused when the cartel wins. Umbrella damages are, therefore, a plausibly first-order phenomenon.

These results provide new evidence on the potential severity of umbrella damages and their ability to broaden the scope of damages caused by a cartel. The recent decision by the ECJ to allow umbrella claimants to pursue damages against cartels seems to recognize the latter fact, albeit pointing at the same time to the difficulty of proving such claims.

Some open questions remain. In particular, the paper focuses on the school milk industry in which contracts are awarded via first-price procurement auctions. Umbrella damages are, however, not confined to auction environments. For instance, Asker, Collard-Wexler, and De Loecker (2019) study global oil extraction and measure the extent of misallocation to non-cartel firms (corresponding to cost inefficiencies due to misallocation in this paper) due to OPEC’s exercise of market power. Investigating the prevalence of such damages in alternative environments is a promising avenue of research and can contribute to emphasizing their importance for rigorous antitrust damage assessment.
References


Tables

Table 1: Number of bids and wins by firm 1980 – 1992

<table>
<thead>
<tr>
<th>Vendor</th>
<th># of bids</th>
<th># of wins</th>
<th>% of wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>BORDEN</td>
<td>836</td>
<td>232</td>
<td>0.278</td>
</tr>
<tr>
<td>CABELL</td>
<td>418</td>
<td>140</td>
<td>0.335</td>
</tr>
<tr>
<td>FOREMOST</td>
<td>139</td>
<td>45</td>
<td>0.324</td>
</tr>
<tr>
<td>GANDY</td>
<td>23</td>
<td>5</td>
<td>0.217</td>
</tr>
<tr>
<td>KNOWLTON</td>
<td>12</td>
<td>1</td>
<td>0.083</td>
</tr>
<tr>
<td>LILLY</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>METZGER</td>
<td>21</td>
<td>4</td>
<td>0.190</td>
</tr>
<tr>
<td>OAK FARMS</td>
<td>530</td>
<td>184</td>
<td>0.347</td>
</tr>
<tr>
<td>PRESTON</td>
<td>333</td>
<td>98</td>
<td>0.294</td>
</tr>
<tr>
<td>PURE</td>
<td>260</td>
<td>73</td>
<td>0.281</td>
</tr>
<tr>
<td>SCHEPPS</td>
<td>528</td>
<td>115</td>
<td>0.218</td>
</tr>
<tr>
<td>SUPERIOR</td>
<td>46</td>
<td>30</td>
<td>0.652</td>
</tr>
<tr>
<td>VANDERVOORT</td>
<td>340</td>
<td>107</td>
<td>0.315</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3,493</strong></td>
<td><strong>1,034</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Descriptive Statistics by Market Area

<table>
<thead>
<tr>
<th>Market Area</th>
<th>DFW</th>
<th>San Antonio</th>
<th>Waco</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bids</td>
<td>2411</td>
<td>555</td>
<td>591</td>
</tr>
<tr>
<td>Number of contracts</td>
<td>735</td>
<td>143</td>
<td>179</td>
</tr>
<tr>
<td>Number of contracts/year</td>
<td>56.5</td>
<td>11</td>
<td>13.7</td>
</tr>
<tr>
<td>Number of counties</td>
<td>25</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Number of school districts</td>
<td>115</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>winning bid/half pint</td>
<td>Mean 0.1442</td>
<td>0.1417</td>
<td>0.1365</td>
</tr>
<tr>
<td></td>
<td>SD 0.0214</td>
<td>0.0283</td>
<td>0.0226</td>
</tr>
<tr>
<td># of meals per year-school district</td>
<td>Mean 522,578.2</td>
<td>1,273,001.4</td>
<td>339,188.0</td>
</tr>
<tr>
<td></td>
<td>SD 655,877.1</td>
<td>1,436,814.3</td>
<td>620,308.4</td>
</tr>
<tr>
<td>Contract total cost per school district</td>
<td>Mean 85,115.37</td>
<td>168,938.34</td>
<td>18,779.90</td>
</tr>
<tr>
<td></td>
<td>SD 112,916.49</td>
<td>202,501.19</td>
<td>24,883.69</td>
</tr>
</tbody>
</table>

*Notes: Bids are for a half-pint of whole white milk. Bids and contract cost are in 1982 dollars.*
Table 3: List of variables

<table>
<thead>
<tr>
<th>Type</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Auction specific</strong></td>
<td>FMO price</td>
<td>raw milk price in 1982 dollars</td>
</tr>
<tr>
<td></td>
<td>meals</td>
<td>number of school lunches per school district per year</td>
</tr>
<tr>
<td></td>
<td>escalated</td>
<td>equals 1 if the bid is escalated</td>
</tr>
<tr>
<td></td>
<td>deliveries</td>
<td>number of deliveries per week</td>
</tr>
<tr>
<td></td>
<td>number of schools</td>
<td>number of schools in the school district</td>
</tr>
<tr>
<td></td>
<td>cooler</td>
<td>equals 1 if a cooler needs to be provided</td>
</tr>
<tr>
<td></td>
<td>number of bids</td>
<td>number of bids submitted in the auction</td>
</tr>
<tr>
<td><strong>Bidder specific</strong></td>
<td>distance</td>
<td>great-circle distance between closest plant and school district</td>
</tr>
<tr>
<td></td>
<td>bid</td>
<td>bid submitted for whole white category</td>
</tr>
<tr>
<td></td>
<td>incumbency</td>
<td>equals 1 if bidder won the contract in previous year</td>
</tr>
</tbody>
</table>

Notes: Indicators for missing values of cooler, deliveries and escalated are also included.

Table 4: Determinants of Pure Milk’s bids

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incumbency</td>
<td>−0.013 (0.015)</td>
<td>−0.029 (0.009)</td>
<td>−0.001 (0.009)</td>
</tr>
<tr>
<td>FMO price</td>
<td>1.175 (0.068)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meals</td>
<td>−0.064 (0.233)</td>
<td>−0.363 (0.141)</td>
<td>−0.339 (0.163)</td>
</tr>
<tr>
<td>Meals (squared)</td>
<td>0.002 (0.010)</td>
<td>0.015 (0.006)</td>
<td>0.013 (0.007)</td>
</tr>
<tr>
<td>Escalated</td>
<td>−0.041 (0.016)</td>
<td>−0.016 (0.010)</td>
<td>−0.019 (0.008)</td>
</tr>
<tr>
<td>Escalated (missing)</td>
<td>−0.015 (0.018)</td>
<td>−0.016 (0.011)</td>
<td>−0.012 (0.010)</td>
</tr>
<tr>
<td>Cooler</td>
<td>0.004 (0.073)</td>
<td>0.007 (0.043)</td>
<td>0.004 (0.056)</td>
</tr>
<tr>
<td>Cooler (missing)</td>
<td>−0.047 (0.067)</td>
<td>−0.017 (0.040)</td>
<td>0.013 (0.053)</td>
</tr>
<tr>
<td>Deliveries</td>
<td>0.004 (0.014)</td>
<td>−0.010 (0.008)</td>
<td>−0.006 (0.009)</td>
</tr>
<tr>
<td>Deliveries (missing)</td>
<td>0.063 (0.051)</td>
<td>0.009 (0.031)</td>
<td>0.040 (0.037)</td>
</tr>
<tr>
<td>Number of schools</td>
<td>0.006 (0.003)</td>
<td>0.003 (0.002)</td>
<td>0.008 (0.003)</td>
</tr>
<tr>
<td>Number of bids</td>
<td>−0.016 (0.013)</td>
<td>−0.016 (0.008)</td>
<td>−0.014 (0.008)</td>
</tr>
<tr>
<td>Distance</td>
<td>0.042 (0.027)</td>
<td>0.080 (0.017)</td>
<td>0.115 (0.022)</td>
</tr>
<tr>
<td>Distance (squared)</td>
<td>−0.005 (0.005)</td>
<td>−0.014 (0.003)</td>
<td>−0.032 (0.006)</td>
</tr>
<tr>
<td>Collusive auction</td>
<td>0.062 (0.021)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.587 (1.372)</td>
<td>0.505 (0.830)</td>
<td>0.631 (0.944)</td>
</tr>
</tbody>
</table>

Year FE: No  Yes  Yes  County FE: No  No  Yes

Observations 217 217 217  R² 0.697 0.908 0.934  Adjusted R² 0.676 0.896 0.921  Residual Std. Error 0.095 (df = 202) 0.054 (df = 191) 0.047 (df = 180)  F Statistic 33.214*** (df = 14; 202) 75.614*** (df = 25; 191) 71.104*** (df = 36; 180)

Notes: A dummy for competitive auctions is omitted in specification (2). The omitted county is BELL in specification (3). All continuous variables in log. Prices in 1982 dollars. FMO price is omitted in specifications (2) and (3) as it varies only by year in the case of Pure Milk’s only plant.
Table 5: Estimates of damages

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean damages per half-pint ($) - LB</td>
<td>0.0033</td>
<td>0.0024 - 0.0077</td>
</tr>
<tr>
<td>Mean damages per half-pint ($) - UB</td>
<td>0.0069</td>
<td>0.0056 - 0.0077</td>
</tr>
<tr>
<td>Mean damages per contract (whole white only) ($) - LB</td>
<td>399.89</td>
<td>293.51 - 946.09</td>
</tr>
<tr>
<td>Mean damages per contract (whole white only) ($) - UB</td>
<td>862.41</td>
<td>661.24 - 958.26</td>
</tr>
<tr>
<td>Mean damages per contract ($) - LB</td>
<td>1,188.29</td>
<td>859.93 - 2,865.21</td>
</tr>
<tr>
<td>Mean damages per contract ($) - UB</td>
<td>2,638.01</td>
<td>1,941.68 - 2,957.66</td>
</tr>
</tbody>
</table>

Mean damages per half-pint ($)

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umbrella damages - LB</td>
<td>0.0019</td>
<td>0.0012 - 0.0075</td>
</tr>
<tr>
<td>Umbrella damages - UB</td>
<td>0.0074</td>
<td>0.0055 - 0.0077</td>
</tr>
<tr>
<td>Cartel damages</td>
<td>0.0056</td>
<td>0.0032 - 0.0112</td>
</tr>
</tbody>
</table>

Mean damages as fraction of competitive winning bid

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umbrella damages - LB</td>
<td>0.0159</td>
<td>0.011 - 0.0666</td>
</tr>
<tr>
<td>Umbrella damages - UB</td>
<td>0.0639</td>
<td>0.0458 - 0.0696</td>
</tr>
<tr>
<td>Cartel damages</td>
<td>0.0494</td>
<td>0.0287 - 0.1011</td>
</tr>
</tbody>
</table>

Mean damages as fraction of competitive winning mark-up

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umbrella damages - LB</td>
<td>0.236</td>
<td>0.1218 - 0.7024</td>
</tr>
<tr>
<td>Umbrella damages - UB</td>
<td>0.6112</td>
<td>0.5014 - 0.6971</td>
</tr>
<tr>
<td>Cartel damages</td>
<td>0.554</td>
<td>0.2991 - 1.0111</td>
</tr>
</tbody>
</table>

Mean damages per contract (whole white only)

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umbrella damages - LB</td>
<td>231.43</td>
<td>149.64 - 905.14</td>
</tr>
<tr>
<td>Umbrella damages - UB</td>
<td>910.67</td>
<td>650.68 - 977.32</td>
</tr>
<tr>
<td>Cartel damages</td>
<td>663.12</td>
<td>389.86 - 1,359.69</td>
</tr>
</tbody>
</table>

Mean damages per contract

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Umbrella damages - LB</td>
<td>701.19</td>
<td>441.94 - 2,722.07</td>
</tr>
<tr>
<td>Umbrella damages - UB</td>
<td>2,760.05</td>
<td>1,865.92 - 3,067.38</td>
</tr>
<tr>
<td>Cartel damages</td>
<td>1,949.39</td>
<td>1,156.52 - 4,157.93</td>
</tr>
</tbody>
</table>

Mean cost inefficiency due to misallocation

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>per half-pint ($)</td>
<td>0.0066</td>
<td>0.0015 - 0.0126</td>
</tr>
<tr>
<td>in percentage loss</td>
<td>0.0594</td>
<td>0.0147 - 0.1272</td>
</tr>
<tr>
<td>per contract ($)</td>
<td>2.337.66</td>
<td>410.02 - 4,824.83</td>
</tr>
</tbody>
</table>

Notes: $ are 1982 dollars. UB corresponds to an inefficient cartel. LB corresponds to an efficient cartel. Only cartel damages in the case of an efficient mechanism are reported. Per contract (whole white only): does not include damages for the other milk categories. Per contract: computed by applying the overcharge estimated to the total quantity purchased (whole white and other categories). Confidence intervals based on 5,000 bootstrap iterations.
Table 6: Damages as fraction of the competitive winning bid (in percent)

**Panel A. Exogenous entry**

<table>
<thead>
<tr>
<th>Bidders</th>
<th>Cartel damages</th>
<th>Umbrella damages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Point Estimate</td>
<td>90% CI</td>
</tr>
<tr>
<td>n₁ n₂ n₃</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 2 0</td>
<td>4.9 3.7 5.9</td>
<td>3.6 2.6 4.7</td>
</tr>
<tr>
<td>1 3 0</td>
<td>6.7 5.1 8</td>
<td>5.7 4.3 7.1</td>
</tr>
<tr>
<td>1 4 0</td>
<td>7.7 5.9 9.2</td>
<td>6.9 5.2 8.4</td>
</tr>
<tr>
<td>1 0 2</td>
<td>4.5 2.9 5.5</td>
<td>3.3 2.2 4.3</td>
</tr>
<tr>
<td>1 0 3</td>
<td>6 3.8 7.5</td>
<td>5.2 3.3 6.6</td>
</tr>
<tr>
<td>1 0 4</td>
<td>6.9 4.3 8.7</td>
<td>6.2 3.9 7.9</td>
</tr>
<tr>
<td>0 3 0</td>
<td>3.9 3.1 4.5</td>
<td>2 1.6 2.3</td>
</tr>
<tr>
<td>0 4 0</td>
<td>5.7 4.5 6.6</td>
<td>4.5 3.6 5.2</td>
</tr>
<tr>
<td>0 5 0</td>
<td>6.8 5.4 8</td>
<td>5.9 4.7 6.8</td>
</tr>
<tr>
<td>0 1 2</td>
<td>3.4 2.6 4.1</td>
<td>1.7 1.2 2.3</td>
</tr>
<tr>
<td>0 1 3</td>
<td>5 3.6 6</td>
<td>3.8 2.8 4.7</td>
</tr>
<tr>
<td>0 1 4</td>
<td>5.9 4.2 7.2</td>
<td>5 3.5 6.2</td>
</tr>
</tbody>
</table>

**Panel B. Endogenous entry**

<table>
<thead>
<tr>
<th>Potential bidders</th>
<th>Cartel damages</th>
<th>Umbrella damages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Point Estimate</td>
<td>90% CI</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N₁ N₂ N₃</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 2 0</td>
<td>2.8 2 3.5</td>
<td>0.8 0.2 1.6</td>
</tr>
<tr>
<td>1 3 0</td>
<td>4.1 3 5.1</td>
<td>2.4 1.5 3.5</td>
</tr>
<tr>
<td>1 4 0</td>
<td>4.9 3.5 6</td>
<td>3.1 2.1 4.3</td>
</tr>
<tr>
<td>1 0 2</td>
<td>2.5 1.6 3.3</td>
<td>0.8 0.2 1.6</td>
</tr>
<tr>
<td>1 0 3</td>
<td>3.7 2.3 4.7</td>
<td>2.2 1.2 3.1</td>
</tr>
<tr>
<td>1 0 4</td>
<td>4.4 2.7 5.6</td>
<td>2.9 1.6 3.9</td>
</tr>
<tr>
<td>0 3 0</td>
<td>2.1 1.5 2.3</td>
<td>-0.9 -1.1 -0.7</td>
</tr>
<tr>
<td>0 4 0</td>
<td>3.1 2.2 3.5</td>
<td>0.7 0.3 0.9</td>
</tr>
<tr>
<td>0 5 0</td>
<td>3.9 2.9 4.4</td>
<td>1.7 1 2</td>
</tr>
<tr>
<td>0 1 2</td>
<td>1.8 1.3 2.2</td>
<td>-0.8 -1.3 -0.1</td>
</tr>
<tr>
<td>0 1 3</td>
<td>2.8 2 3.3</td>
<td>0.6 0 1.1</td>
</tr>
<tr>
<td>0 1 4</td>
<td>3.5 2.5 4.2</td>
<td>1.5 0.8 2.1</td>
</tr>
</tbody>
</table>

*Notes:* Mean damages are averaged across collusive auctions in which Pure Milk bid against the cartel. The variables \((n₁, n₂, n₃)\) (resp. \((N₁, N₂, N₃)\)) correspond to the number of \{small \times near\}, \{large \times far\}, and \{large \times near\} bidders (resp. potential bidders). The thresholds used to define firm types are 70 miles for distance and the median firm capacity for firm size. Capacity refers to the maximum total dollar value won by the firm in any year over competitive and rigged school districts. The cartel mechanism is assumed efficient. All estimates are conditional on the cartel and the non-cartel bidder entering the collusive auction. Confidence intervals based on 5,000 bootstrap iterations.
Figures

Figure 1: Map of the counties in the dataset, by market area
Figure 2: Estimates and 95% Confidence Intervals for the county dummies
Figure 3: Estimated bid functions for auctions with three bidders with an efficient and inefficient cartel mechanism

(a) Efficient cartel mechanism

(b) Inefficient cartel mechanism
A Theoretical results

A.1 Effect of collusion on the non-cartel firm’s profits

This subsection examines the effect of collusion between a strict subset of bidders on the non-cartel firm’s profits. I show that the non-cartel firm’s interim payoff (i.e., the expected profits conditional on its cost) always increases when its competitors form a cartel.

Define the outsider’s equilibrium interim payoff when facing \(n\) competitors (no collusion):

\[
\pi(c) = \max_b (b - c) \left[1 - F(\phi(b))\right]^n
\]

where \(F\) denotes the symmetric cost distributions for all bidders. Similarly, define the outsider equilibrium interim payoff when facing the cartel bidder (the \(n\) competitors collude):

\[
\tilde{\pi}(c) = \max_b (b - c) \left[1 - F_1(\phi_1(b))\right]
\]

where \(F_1\) is the cost distribution of the cartel bidder. Assume first that the cartel mechanism is efficient, i.e. \(F_1(c) = 1 - (1 - F(c))^n\). Then, one obtains the following lemma:

**Lemma 2.** If the cartel mechanism is efficient, the outsider’s interim payoff is strictly larger when its competitors collude:

\[
\tilde{\pi}(c) > \pi(c) \quad \text{for all } c \in [c, \bar{c})
\]

**Proof.** Note that if \(\phi\) solve the symmetric \(n + 1\) bidders procurement auction, it also solves the two bidder procurement auction in which bidders’ cost are drawn from \(1 - (1 - F(c))^n\). When the \(n\) firms collude efficiently, we obtain a two-bidder asymmetric auction in which bidders’ costs are drawn from \(F\) for the non-cartel firm and \(1 - (1 - F(c))^n\) for the cartel bidder. Because \(F\) and \(1 - (1 - F(c))^n\) can be ordered stochastically in the hazard rate sense, Corollary 1 of Lebrun (1998) applies to obtain the strict inequality for the non-cartel firm profits.

In general, the cartel mechanism might not be efficient. Assume that the cartel bidder’s cost distribution \(F_1\) satisfies the *conditional stochastic dominance relation*\(^{56}\)

\[
\frac{d}{dc} \left(\frac{1 - F_1(c)}{(1 - F(c))^n}\right) > 0 \quad \text{for all } c \in (\underline{c}, \bar{c}]
\]  

\(^{56}\)Note that this relation can also be rewritten in terms of hazard rate dominance.
Proposition 2. If (19) holds, the outsider’s interim payoff is strictly larger when its competitors collude:

\[ \tilde{\pi}(c) > \pi(c) \quad \text{for all } c \in [\underline{c}, \overline{c}] \]

Proof. \( F_1 \) and \( 1 - (1 - F(c))^n \) satisfy the assumption of conditional stochastic dominance, therefore, Corollary 1 in Lebrun (1998) implies that the outsider’s interim payoff is strictly larger when facing the cartel with the mechanism yielding \( F_1 \) than when facing an efficient cartel. Combining this observation with the previous lemma gives the result. \( \Box \)

A.2 Effect of the cartel size and cartel mechanism on the non-cartel firm’s bidding

In this subsection, comparative statics for the non-cartel firm equilibrium bid function are presented. In particular, two features are investigated: the cartel size and the cartel mechanism. Comparative statics in first-price asymmetric auctions have been studied by Lebrun (1998). The results presented here are an application of the main theorem proved in the latter paper to the specific case of a cartel bidding in a procurement auction.

First, let the cartel mechanism be fixed. Using the same notations introduced above, denote by 1 the cartel bidder, and by 2 the non-cartel bidder (\( F_i \), for \( i \in \{1, 2\} \) the corresponding cost distributions, which satisfy the assumptions of the model). The dependency of the cartel bidder’s cost distribution on the cartel size \( n \) is explicitly represented as: \( F_1(\cdot|n) \). In particular under Assumption \( 2 \) if the cartel is efficient: \( F_1(c|n) = 1 - (1 - F(c))^n \) (minimum cost among \( n \) symmetric bidders). If the cartel is inefficient: \( F_1(c|n) = F(c) \) (cartel bidder selected randomly).

Assume the cartel mechanism is such that \( F_1(\cdot|n) \) satisfies the following conditional stochastic dominance condition

\[ \frac{d}{dc} \left( \frac{1 - F_1(c|n)}{1 - F_1(c|n + 1)} \right) > 0 \quad \text{for all } c \in (\underline{c}, \overline{c}] \quad (17) \]

Proposition 3. Assume that (17) holds. Let the bid functions and their inverses at the unique equilibrium when the cartel size is \( n \) be denoted \( \beta_1(\cdot|n), \beta_2(\cdot|n) \) and \( \phi_1(\cdot|n), \phi_2(\cdot|n) \) respectively. Then

1. As the cartel size increases, the non-cartel firm bids more aggressively:

\[ \beta_2(c|n) > \beta_2(c|n + 1) \quad \text{for all } c \in [\underline{c}, \overline{c}] \]
2. The larger cartel’s bid distribution is stochastically dominated by the smaller cartel’s bid distribution:

\[ F_1(\phi_1(b|n)|n) < F_1(\phi_1(b|n+1)|n + 1) \quad \text{for all } c \in [b, c) \]

where \( b = \beta_1(c|n) = \beta_2(c|n) \).

3. As the cartel size increases, both the cartel and non-cartel bidders’ interim profits (conditional on their private cost) decrease.

Proof. Follows directly from Theorem 1 and Corollary 1 in Lebrun (1998).

Condition (17) holds when the cartel mechanism is efficient. However, the condition does not hold if the cartel mechanism is inefficient (\( F_1 \) is independent of \( n \)).

Next, let the cartel size be fixed. The effect of the cartel mechanism on the non-cartel firm bidding can be analyzed. Consider two mechanisms implying a cartel bidder’s cost distribution of either \( F_1 \) or \( \tilde{F}_1 \). Assume the two distributions satisfy the conditional stochastic dominance relation:

\[ \frac{d}{dc} \left( \frac{1 - F_1(c)}{1 - \tilde{F}_1(c)} \right) > 0 \quad \text{for all } c \in [c, \bar{c}] \] (18)

This condition implies in particular that \( \tilde{F}_1 \) is first order stochastically dominated by \( F_1 \). For instance, if \( F_1(c) = F(c) \) (inefficient cartel) and \( \tilde{F}_1(c) = 1 - (1 - F(c))^n \) (efficient cartel), the condition holds.

Proposition 4. Assume (18) holds. Let the bid functions and their inverses at the unique equilibrium when the cartel bidder’s distribution is \( F_1 \) (resp. \( \tilde{F}_1 \)) be denoted \((\beta_1, \beta_2)\) (resp. \((\tilde{\beta}_1, \tilde{\beta}_2)\)) and \((\phi_1, \phi_2)\) (resp. \((\tilde{\phi}_1, \tilde{\phi}_2)\)). Then

1. The non-cartel firm bids more aggressively when the cartel bidder’s cost distribution is \( \tilde{F}_1 \) than when it is \( F_1 \):

\[ \beta_2(c) > \tilde{\beta}_2(c) \quad \text{for all } c \in [c, \bar{c}] \]

2. The cartel’s equilibrium bid distribution can be ordered according to first-order stochastic dominance:

\[ F_1(\phi_1(b)) < \tilde{F}_1(\tilde{\phi}_1(b)) \quad \text{for all } c \in [b, \bar{c}] \]

where \( b = \beta_1(c) = \beta_2(c) \)

3. Cartel and non-cartel bidders’ interim profits (conditional on their private costs) are lower under \( \tilde{F}_1 \) than under \( F_1 \).
Proof. Follows directly from Theorem 1 and Corollary 1 in Lebrun (1998).

The intuition for Proposition 3 and 4 is as follows. In both cases, as the cartel bidder is made stronger (either by increasing the size of the cartel or by making the cartel mechanism more efficient), the non-cartel firm responds by bidding more aggressively (part (1) of the propositions). As a consequence, the cartel bidder’s interim payoff decreases. In equilibrium, the cartel best response is such that it is more likely to bid lower: the new equilibrium bid distribution is first-order stochastically dominated by its initial equilibrium bid distribution (part (2) of the propositions). This results in the non-cartel firm interim payoff being lower as well.

A.3 Characterization of bounds on damages

This subsection investigates the effect of the cartel mechanism on the size of damages. Damages can be ordered according to the efficiency of the cartel mechanism. I show, in particular, that efficient and inefficient mechanisms provide bounds on damages within a simple class of cartel mechanisms.

A cartel mechanism is defined as a (possibly stochastic) rule that selects a cartel member to bid in the target auction. As in McAfee and McMillan (1992), this definition abstracts from the particular enforcement mechanism used by the cartel (e.g., repeated interactions or the use of an enforcer). I also abstracts from modelling transfers between members. As a shorthand, in the rest of this subsection, a cartel mechanism refers to the cost distribution of the selected cartel bidder.

I restrict the analysis to a class of cartel mechanisms satisfying the following two conditions: First, the mechanism does not dictate bidding in the target auction, i.e., the cartel bidder bids to maximize her expected profits given her private cost (in accordance with the model of Section 2). This assumption rules out mechanisms where side-payments distort bidding in the target auction, as in the case studied by Asker (2010). In his study comparing the Texas and the Florida milk cartels, Pesendorfer (2000) finds evidence consistent with the Texas cartel operating without side-payments. Second, I assume that the cost distribution of the selected cartel bidder is of the form

\[ F_1(c) = 1 - (1 - F(c))^\theta \quad \text{for} \quad \theta \in [1, \infty) \]

where \( F \) denotes the symmetric cost distributions for all cartel members. This second condition guarantees that all cartel mechanisms in the class studied can be stochastically ordered in the hazard rate sense. With \( n \) cartel members, \( \theta \) can be at most equal to \( n \).
Denote by $\beta$ the symmetric equilibrium bidding strategy in the auction where the non-cartel firm faces $n$ non-colluding bidders. Let $b_{w}^{\text{comp}}$ denote the winning bid in the competitive auction

$$b_{w}^{\text{comp}} = \min \left( \{ \beta(c_i) \}_{i=1}^{n+1} \right)$$

In the collusive auction, denote by 1 the cartel bidder and by 2 the non-cartel bidder. Consider two mechanisms, from the class described above, implying a cartel bidder’s cost distribution of either $F_1$ or $\tilde{F}_1$. Assume the two distributions satisfy the conditional stochastic dominance relation\(^{57}\)

$$\frac{d}{dc} \left( \frac{1 - F_1(c)}{1 - \tilde{F}_1(c)} \right) > 0 \quad \text{for all } c \in (\underline{c}, \overline{c}]$$  \hspace{1cm} (19)

This condition implies in particular that $\tilde{F}_1$ is first-order stochastically dominated by $F_1$. For instance, the condition holds if $F_1(c) = F(c)$, i.e., the cartel is inefficient and the cartel bidder is selected randomly ($\theta = 1$) and if $\tilde{F}_1(c) = 1 - (1 - F(c))^n$, i.e., the cartel is efficient and the cartel bidder is the lowest cost member ($\theta = n$).

**Proposition 5.** Assume (19) holds. Let the bid functions at the unique equilibrium when the cartel bidder’s distribution is $F_1$ (resp. $\tilde{F}_1$) be denoted $(\beta_1, \beta_2)$ (resp. $(\tilde{\beta}_1, \tilde{\beta}_2)$). Let $b_{w}^{\text{col}}$ and $\tilde{b}_{w}^{\text{col}}$ denote the winning bids in the collusive auction under mechanism $F_1$ and $\tilde{F}_1$ respectively. Then

1. The total expected damages to the auctioneer are larger when the cartel bidder’s cost distribution is $F_1$ than when it is $\tilde{F}_1$:

$$E \left[ b_{w}^{\text{col}} - b_{w}^{\text{comp}} \right] > E \left[ \tilde{b}_{w}^{\text{col}} - b_{w}^{\text{comp}} \right]$$

2. The expected damages to the auctioneer, conditional on the non-cartel firm having the lowest cost, are larger when the cartel bidder’s cost distribution is $F_1$ than when it is $\tilde{F}_1$:

$$E \left[ \beta_2(c_2) - \beta(c_2) \mid \beta(c_2) = b_{w}^{\text{comp}} \right] > E \left[ \tilde{\beta}_2(c_2) - \beta(c_2) \mid \beta(c_2) = b_{w}^{\text{comp}} \right]$$

**Proof.** Part 1 of the proposition is a consequence of results 1 and 2 in Proposition 4. Both the cartel and non-cartel bidder’s bid distributions under $F_1$ stochastically dominate their bid distributions under $\tilde{F}_1$. Therefore, the distribution of the collusive winning bid under $F_1$ (denoted $b_{w}^{\text{col}}$) stochastically dominates the distribution of the collusive winning bid under $\tilde{F}_1$.

\(^{57}\)This relation is equivalent to the hazard rate dominance. Within the class of mechanisms studied, if $F_1$ and $\tilde{F}_1$ are parameterized by $\theta$ and $\tilde{\theta}$, this condition is equivalent to $\theta < \tilde{\theta}$. 

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$F_1$ (denoted $\tilde{b}^\text{col}_w$). As the expected competitive winning bid is independent of the cartel mechanism, the total expected damages are larger under $F_1$.

Part 2 of the proposition is a consequence of result 1 in Proposition \ref{prop2}. Conditional on being the lowest cost bidder, the non-cartel firm bids more aggressively when facing a cartel bidder with distribution $\tilde{F}_1$ than with distribution $F_1$, that is,

$$\beta_2(c_2) > \tilde{\beta}_2(c_2)$$

As the expected competitive winning bid is independent of the cartel mechanism (conditional on the non-cartel firm being the lowest bidder, that is, $\beta(c_2) = b^{\text{comp}}_w$), expected damages are larger under $F_1$.

Any mechanism in the class defined above, such that $\theta < n$, generates a cost distribution for the cartel bidder that is stochastically dominated in the hazard rate sense by the cost distribution under an efficient cartel mechanism ($\theta = n$). Proposition \ref{prop5} implies that damages under an efficient cartel mechanism provide a lower bound on total damages and damages conditional on the non-cartel firm being the lowest cost bidder.

To obtain an upper bound on damages, I assume that the cartel is able to select the cartel bidder at least as well as at random, i.e., $\theta \geq 1$. This is consistent with the findings of Pesendorfer (2000) who shows that although the Texas cartel did not use side-payments, its functioning was almost efficient thanks to the large number of contracts to be divided. If $\theta \geq 1$, Proposition \ref{prop5} implies that damages to the auctioneer are highest with an inefficient cartel.

The bounds obtained are sharp in instances where plaintiffs purchased from both cartel and non-cartel firms, as was the case for school districts in DFW. Indeed, civil cases would aim to assess the total amount of damages (inflicted by both cartel and non-cartel firms). If a plaintiff purchased only from non-cartel firms, the bounds obtained are no longer sharp because only damages when the non-cartel firm is the lowest cost bidder can be bounded. If the non-cartel firm has a higher cost than the cartel bidder but still wins the asymmetric collusive auction, damages cannot be ordered. In theory, misallocation due to asymmetries has an ambiguous effect on the size of umbrella damages: in particular, the probability that the contract is mis-allocated to the non-cartel firm can be non-monotone in the parameter $\theta$.

\footnote{When assessing damages for recovery to affected schools, the proposed approach allows to bound the total damages including both direct cartel and umbrella damages.}
B Algorithm for solving the asymmetric auction (Step 3)

The details of the numerical method used to solve the asymmetric auctions are presented here. Recall that the cartel and non-cartel bidders’ equilibrium inverse bid functions, denoted \( \phi_1 \) and \( \phi_2 \) respectively, are the solutions of:

\[
\begin{align*}
\frac{d\phi_1}{db} & = \frac{1-F_1(\phi_1(b))}{f_1(\phi_1(b))} \frac{1}{(b-\phi_2(b))} \\
\frac{d\phi_2}{db} & = \frac{1-F_2(\phi_2(b))}{f_2(\phi_2(b))} \frac{1}{(b-\phi_1(b))}
\end{align*}
\]  

(20)

where \( F_i \) (resp. \( f_i \)) is the cumulative distribution (resp. density function) of costs of the cartel \((i = 1)\) and the outsider firm \((i = 2)\). Along with the boundary conditions \( \phi_1(b) = \phi_2(b) = \zeta \) (for some \( b \), lower bound of the support of bids) and \( \phi_1(\bar{c}) = \phi_2(\bar{c}) = \bar{c} \). The location of the left-boundary \( b \) is unknown. As shown in Fibich and Gavish (2011), the system of differential equations (20) can be recast as a system in \( \phi_1 \) and \( b \) (instead of \( \phi_2 \)) as functions of \( \phi_2 \). After this change of variable, the new BVP is given by:

\[
\begin{align*}
\frac{d\phi_1}{d\phi_2} & = \frac{1-F_1(\phi_1)}{f_1(\phi_1)} \frac{f_2(\phi_2)}{1-F_2(\phi_2)} \frac{b-\phi_1}{b-\phi_2} \\
\frac{db}{d\phi_2} & = \frac{f_2(\phi_2)}{1-F_2(\phi_2)} (b - \phi_1)
\end{align*}
\]

(21)

with the boundary conditions: \( \phi_1(\phi_2 = \bar{c}) = b(\phi_2 = \bar{c}) = \bar{c} \), and \( \phi_1(\phi_2 = \zeta) = \zeta \). This BVP is defined on a known domain \( \phi_2 \in [\zeta, \bar{c}] \). Fibich and Gavish (2011) propose fixed point iterations as one possible method for solving (21). Iterations are given by:

\[
\begin{align*}
\frac{d\phi_1}{d\phi_2} & = \frac{1-F_1(\phi_1^{(k)})}{f_1(\phi_1^{(k)})} \frac{f_2(\phi_2^{(k)})}{1-F_2(\phi_2^{(k)})} \frac{b^{(k)}-\phi_1^{(k)}}{b^{(k)}-\phi_2^{(k)}} \\
\frac{db}{d\phi_2} & = -\frac{b^{(k)}-\phi_1^{(k+1)}}{b^{(k)}-\phi_2^{(k)}} \frac{f_2(\phi_2^{(k)})}{1-F_2(\phi_2^{(k)})} \phi_2^{(k+1)}
\end{align*}
\]

(22)

with the boundary conditions: \( \phi_1^{(k+1)}(\phi_2 = \bar{c}) = b^{(k+1)}(\phi_2 = \bar{c}) = \bar{c} \), and \( \phi_1^{(k+1)}(\phi_2 = \zeta) = \zeta \). In the case of this particular empirical application, the initial guess used are: \( \phi_1^{(0)}(\phi_2) = \phi_2 \) and \( b^{(0)}(\phi_2) = 0.9 + (\bar{c} - 0.9)/\bar{c} \times \phi_2 \). Although convergence of the fixed point iterations is not guaranteed, because a unique solution exists, if the algorithm converges to a function satisfying the BVP, this function is the solution to the BVP.