A Study of Umbrella Damages from Bid-Rigging

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Abstract

If non-cartel firms adjust their pricing to the supra-competitive level sustained by a cartel, purchasers from non-cartel firms may suffer umbrella damages. This article examines the bidding behavior of non-cartel firms bidding against the Texas school milk cartel between 1980 and 1992. I find that the largest non-cartel firm bid significantly higher when facing the cartel. Structural estimation reveals that, per contract, damages due to non-cartel firms overbidding are at least 47% of damages caused by the cartel. Inefficiencies raise the winner’s cost by 3.7%. These results shed light on the importance of umbrella damages from a civil liability perspective.

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1 Introduction

When a cartel does not include every firm competing in an industry, non-cartel firms can set their prices higher than they would otherwise have been able to under competitive conditions. This occurs, in particular, in markets where contracts are awarded via first-price procurement auctions. When the results of such procurement procedures (bids and bidders identities) are not concealed from non-cartel firms, they may serve as an indication of the prevailing price level when future contracts are procured for. Consequently, non-cartel bidders benefit from the protection of the cartel’s inflated bidding, and operate “under the cartel’s umbrella.” Purchasers from non-cartel bidders will still pay a price that exceeds what the market price would be in the absence of collusion. In this sense, damages inflicted by non-cartel bidders broaden the scope of cartel damages. Nonetheless, empirical research investigating the importance of such damages remains scarce. This article conducts a detailed study of umbrella damages by examining the bidding behavior of non-cartel bidders facing the Texas school milk cartel between 1980 and 1992.

Bid-rigging was common in auctions for the supply of milk to schools, at least until the early 1990s. According to Porter and Zona (1999), investigations were conducted in more than twenty states across the US, more than $90 million of fines were imposed, while about 90 people were sent to jail for sentences lasting 6 months on average. The Texas milk cartel is well-suited for analyzing damages inflicted by non-cartel bidders. First, two essential conditions are met: the cartel was not all-inclusive, and the auction format was first-price sealed bid. In particular, bids and bidder identities are publicly announced. This public information enables non-cartel firms to learn and adjust their bids to the supra-competitive levels sustained by the cartel. Second, all firms involved in the cartel were convicted, which allows me to isolate non-cartel bidders and focus on their bidding behavior. Third, the dataset collected by the Antitrust Division of the Department of Justice spans markets with and without cartel operations, which enables to identify the effect of the cartel’s price umbrella on non-cartel bidders. Finally, the dataset is rich enough to allow an assessment of damages inflicted by the cartel per se, as well as damages inflicted through non-cartel bidders.

Reduced form analysis of the bid data reveals that the largest non-cartel firm bid significantly higher when facing the cartel, controlling for auction and bidder observed het-

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1 In ascending or second price auctions, bidding one’s private valuation is a dominant strategy irrespective of the existence of the cartel; therefore, umbrella damages do not arise. For umbrella damages to arise in first-price procurement auction, one need not assume that non-cartel firms know the existence of the cartel per se, but only that they know the equilibrium bid distribution of their lowest bidding opponent, which can be inferred from past auctions.
erogeneity. Further investigation of cartel and umbrella damages and inefficiencies requires
the estimation of a structural model. Damages to the auctioneer are decomposed into (1) *cartel damages*, defined as damages in auctions won by the cartel and (2a) *outsider damages*, defined as damages in auctions won by the non-cartel firm when it is the lowest cost bidder and (2b) *misallocation damages*, defined as damages in auctions won by the non-cartel firm when the cartel bidder is the lowest cost bidder. Case (2b) arises because partial collusion introduces asymmetry among bidders in the first-price procurement setting: the cartel bidder has a stronger incentive to inflate his bid above his cost than the non-cartel bidder. As a result, the winner is not necessarily the lowest cost bidder, and the auction is no longer efficient. (2a) and (2b) form what is defined as umbrella damages. Because the cartel’s internal structure is unknown, bounds on outsider and misallocation damages are derived assuming two extreme cases for the cartel mechanism. Either the mechanism is efficient (i.e., the lowest cost cartel member is the cartel bidder in the target auction), or the mechanism is inefficient (i.e., the cartel bidder is selected randomly among cartel members).

The structural analysis shows that per contract, outsider damages (conditional on the non-cartel firm winning) are at least 47% of cartel damages (conditional on the cartel winning). This lower bound is obtained with an efficient cartel. If the cartel is inefficient, outsider damages can be as large as cartel damages. Misallocation damages are estimated to be as large as 64% when the cartel is efficient, and the auction is asymmetric. Concerning inefficiencies due to the asymmetry across bidders, losses are estimated to be 3.7% or $2,909 per contract. The structural estimates show that, conditional on the non-cartel firm winning, prices are inflated by 2.9% to 8.5% relative to the competitive winning bid. These bounds are consistent with the reduced form estimates of a 6% overcharge. These results point to a cartel mechanism that was not entirely efficient, but far from inefficient.

From a competition law perspective, US and European civil courts have historically not recognized the ability of purchasers from non-cartel firms to pursue colluders for umbrella damages, the reason being the complexity of proving umbrella claims. This article sheds new light on this debate by providing a case study of umbrella damages, emphasizing the type of data and methodology that render the estimation of damages possible and far from speculative. The US and European competition laws have recently taken divergent paths

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2Whereas the definition of an efficient cartel mechanism is straightforward, it is less evident for an inefficient cartel mechanism. In the inefficient case, the cartel bidder could be randomly selected as is assumed in this article. However, one could think of other inefficient mechanisms such as the one in which the cartel bidder is the least cost-efficient member.

3If the cartel is inefficient, there is no asymmetry between bidders under the maintained assumptions and therefore no efficiency losses.

4This is consistent with Pesendorfer (2000) which shows that although the cartel did not use side-payments, it managed to retain quasi-efficient collusive rents through market division.
in their treatment of whether the civil liability in damages of the cartel members extends to umbrella damages. On June 5\textsuperscript{th} 2014, the European Court of Justice handed down the awaited judgment in the elevator cartel case (\textit{Kone AG and Others v. ÖBB-Infrastruktur AG}), stating the right of plaintiffs to compensation for umbrella damages\textsuperscript{5} In its press release No 79/14, the ECJ states:

[...] where it has been established that the cartel is, in the circumstances of the case and, in particular, the specific aspects of the relevant market, liable to result in prices being raised by competitors not a party to the cartel, the victims of this price increase must be able to claim compensation for loss sustained from the members of the cartel.

At the same time, the ECJ emphasizes the ”high hurdles in terms of the burden of proof that await” any umbrella claimants\textsuperscript{6}

In the US, competition law is inconsistent in its standing vis-a-vis umbrella claims. The ability of purchasers from non-cartel firms to recover treble damages from the conspirators under section 4 of the Clayton Act is uncertain because of the Supreme Court decision in \textit{Illinois Brick Co. v. Illinois}\textsuperscript{7} In the case, the Court ruled that indirect purchasers (downstream buyers) may not sue on a theory that a price-fixing overcharge has been passed on to them by intermediate sellers purchasing from upstream colluders. This case was relied on by courts to deny standing to purchasers from non-cartel firms\textsuperscript{8} Again, one of the policy reasons underlying the \textit{Illinois Brick} doctrine revolves around the complexity of tracing out the causal link between the antitrust violation and non-cartel firms’ response\textsuperscript{9} This article shows that in the particular case of bid-rigging of procurement auctions, bid data

\textsuperscript{5}The elevator cartel, which involved the conclusion of anticompetitive agreements between major European manufacturers of elevators and escalators (Kone, Otis, Schindler, and ThyssenKrupp) operated in several Member States of the European Union over many years. The European Commission uncovered that cartel in 2003 and, in 2007, imposed fines for the elevator cartel’s practices in the Belgian, German, Netherlands and Luxembourg markets.

\textsuperscript{6}Opinion of the Advocate General Kokott.

\textsuperscript{7}Section 4 of the Clayton Act provides that ”any person ... injured in his business or property” by an antitrust violation may bring an action for treble damages.


\textsuperscript{9}Many states, including California, have enacted Illinois Brick-repealer legislation providing indirect purchasers standing to sue for antitrust violations.
can be leveraged to estimate the size of damages when buyers do not contract directly with colluders.

This article also relates to the debate around auction format choice. A commonly advanced argument in favor of sealed bid auctions is that open auctions are more prone to collusion because conspirators can immediately punish any deviations. This reasoning does not account for the fact that given (partial) collusion, the auctioneer will suffer damages of a greater scope in sealed bid auctions. As non-cartel firms adjust their bidding strategy to the cartel, damages to the auctioneer extend to contract won by non-conspiring firms. Whereas in ascending auctions, the auctioneer suffers damages only when the cartel can suppress the second highest bid (the auctioneer may benefit in some cases from the cartel’s overbidding as found by Asker (2010)). This work shows that a buyer from non-conspiring firms can suffer damages of a similar magnitude than if they contracted directly with conspiring firms. The potential merit of sealed bid relative to open auctions is therefore nuanced with regard to these findings.

The US milk cartels were the subject of various articles in the empirical literature on bid-rigging. Pesendorfer (2000) examines the Florida and Texas school milk cartels and shows that the data (in particular market shares and incumbency rates) is consistent with a strong cartel in Florida (in the sense that side-payments were used between cartel members) and a weak cartel in Texas (no side-payments). This article differs from the former in two dimensions: first, Pesendorfer focuses on the Dallas-Fort Worth (DFW) area whereas the dataset used in this article includes, in addition, the Waco and San Antonio areas in which the cartel was not operating, providing a set of competitive auctions which are useful to predict counterfactuals; second, this article focuses primarily on outsiders’ response to the cartel’s bidding behavior. Hewitt, McClave, and Sibley (1996) demonstrate that the high incumbency rates in Texas (the supplier of a given school district does not change from year to year in many cases) can only be explained by collusion. Lee (1999) finds evidence of complementary bidding and high incumbency premia in the DFW school milk market. Porter and Zona (1999) test for the presence of collusion in the Ohio school milk market by comparing defendants firms in Cincinnati to a control group of non-defendants and compute estimates of cartel damages. Lanzillotti (1996) provides a review of US milk cartel cases and shows that several features of the bids in Kentucky are indicative of collusive behavior.


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10 See the discussion in Athey, Levin, and Seira (2011) for timber auctions.
for timber auctions. A second strand in the literature focuses on the internal organization of bidding rings. Asker (2010) studies equilibrium bidding and side-payments in knock-out auctions held privately by the New York stamp cartel before the actual auction. Kwoka Jr (1997) analyzes bids and side-payments in knock-out auctions held by a real estate ring. Finally, a more recent strand of the literature, closer to this article, studies the interaction of partial cartels and non-cartel bidders. Harrington, H"uschelrath, and Laitenberger (2016) analyzes how the German cement cartel controlled the expansion of non-cartel supply (from Eastern European countries) by sharing the collusive rents with German importers.

The empirical section of the article relies on results from the literature on the structural estimation of auctions. In particular, the non-parametric estimation of the bidders’ underlying cost distribution follows the methodology introduced in Guerre, Perrigne, and Vuong (2000). Observed auction heterogeneity is also controlled for by using the first-stage regression technique developed in Haile, Hong, and Shum (2006). Finally, the empirical section makes use of some of the numerical methods developed in the computational literature on asymmetric auctions. Recent contributions in this literature are: Li and Riley (2007), Gayle and Richard (2008), and Fibich and Gavish (2011).

The next section describes the theoretical model of first-price procurement auctions with asymmetric bidders and analyzes the effect of collusion, cartel size, and cartel mechanism on the non-cartel firm profits and bidding behavior. Section 2 presents the school milk market and the relevant factors affecting the cost structure. Section 3 describes the dataset. Section 4 examines the largest non-cartel firm’s bidding behavior through a reduced-form approach, and shows that all else equal, the largest non-cartel bidder overbids in school district where the cartel is operating. An assessment of cartel and umbrella damages is presented in section 5 using a structural approach.

2 A Theoretical Model

This section investigates properties of a non-cartel firm’s bidding behavior when facing a cartel bidder. The section builds on the theoretical literature on asymmetric first price auctions. In particular, Maskin and Riley (2000) and Lebrun (1999, 2006) derive existence and uniqueness results, as well as comparative statics results for the equilibrium bid functions. This section is also related to the theoretical literature on bidding rings in first-price auctions: McAfee and McMillan (1992) characterize the optimal mechanism for strong and

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11 The cartel bidder is defined as the cartel member selected to bid in the auction on behalf of the cartel.

weak cartels, whereas Marshall and Marx (2007) compares first and second price auction formats when a cartel is not all-inclusive and may or may not be able to control the bids of its members.

The questions of interest are: when does non-cartel firms benefit from their competitors colluding? How are non-cartel firm’s bidding and profits affected by the cartel size and the cartel mechanism (relative to a situation with no cartel)?

The case of a single non-cartel firm is considered for simplicity, and in anticipation of the empirical part. Risk-neutral firms are bidding for a single contract in a first-price procurement setting. There is no reserve price. Denote by 1 the cartel bidder, and by 2 the non-cartel bidder. For $i \in \{1, 2\}$, firm $i$’s cost $c_i$ is drawn from a distribution $F_i$ with support $[c, \bar{c}]$. $F_i$ has a continuous density $f_i$ strictly positive on $(c, \bar{c}]$. Cost are drawn independently across bidders, and are private. Existence and uniqueness of an equilibrium in strictly increasing strategies was proved by Maskin and Riley (2000) and Lebrun (1996, 1999, 2006) among others.

Denote by $\beta_i$ bidder $i$’s equilibrium bidding strategy, and by $\phi_i = \beta_i^{-1}$ the corresponding inverse bid function. Note that if bidder $j$ bids according to $\phi_j$ and bidder $i$ submits a bid $b$, then the latter wins if and only if $c_j > \phi_j(b)$. Thus, bidder $i$’s expected profit from bidding $b$ is given by:

$$\pi(b; c_i) = (b - c_i) \Pr(c_j > \phi_j(b)) = (b - c_i) (1 - F_j(\phi_j(b)))$$

The first-order condition with respect to $b$ (at $c_i = \phi_i(b)$) is:

$$f_j((\phi_j(b)) \phi_j'(b) = \frac{1}{b - c_i} \text{ with boundary condition } \phi_i(\bar{c}) = \bar{c}.$$  

Combining optimality conditions for the two bidders, the equilibrium bid functions solve the following system of differential equations:

$$\begin{cases}
\frac{1}{b - \phi_1(b)} = \frac{f_2(\phi_2(b))}{1 - F_2(\phi_2(b))} \phi_2'(b) \\
\frac{1}{b - \phi_2(b)} = \frac{f_1(\phi_1(b))}{1 - F_1(\phi_1(b))} \phi_1'(b)
\end{cases}$$

with right-boundary conditions : $\phi_1(\bar{c}) = \phi_2(\bar{c}) = \bar{c}$.

Maskin and Riley (2000) show that inverse bid functions must satisfy the additional condition that the minimum bid of all bidders is the same: $\phi_1(b) = \phi_2(b) = c$ for some unknown $b$. Although analytical solutions of problem (3) are in general not available, one can derive properties on the equilibrium bid

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13Note that this setting does not rule out the possibility that other cartel members submit "non-serious" or "complementary" bids. If that is the case, the non-cartel firm knows that it is facing only one "serious" bid from the cartel.
functions.

The first property, proved by Maskin and Riley (2000) and Pesendorfer (2000), can be used to compare the cartel and non-cartel firms bid functions. Assume that the bidders’ cost distributions can be ordered according to hazard rate dominance, i.e.

\[
\frac{f_1(c)}{1 - F_1(c)} > \frac{f_2(c)}{1 - F_2(c)} \quad \text{for all } c
\]

(4)

This implies that, conditional on having a cost above \(c\), the cartel bidder is more likely to have a low cost than the non-cartel bidder.

**Proposition 1.** Under the hazard rate dominance assumption:

1. The cartel firm bids higher than the non-cartel firm: \(\beta_1(c) > \beta_2(c)\) for all \(c \in (\xi, \bar{c})\)

2. Denoting by \(G_i\) bidder \(i\)’s equilibrium bid distribution (and \(g_i\) the corresponding density function), the cartel firm’s bid distribution dominates the non-cartel firms bid distribution (in the hazard rate sense):

\[
\frac{g_1(b)}{1 - G_1(b)} > \frac{g_2(b)}{1 - G_2(b)}
\]

**Proof.** See Propositions 3.3 and 3.5 in Maskin and Riley (2000) or Krishna (2006). \(\square\)

Part 1 of the proposition implies that the non-cartel firm may win despite not having the lowest cost among the bidders. This result in an inefficient allocation. Part 2 of the proposition implies in particular that the non-cartel firm’s equilibrium bid distribution first-order stochastically dominates the cartel firm’s equilibrium bid distribution. As noted by Pesendorfer (2000), the hazard rate dominance condition will be satisfied for instance when all firms (cartel and non-cartel firms) are ex-ante symmetric and the cartel mechanism is efficient, i.e when the cartel bidder has the lowest cost among cartel members. In this case, \(F_1(c) = 1 - (1 - F(c))^n\) where \(F\) is the ex-ante symmetric distribution of bidders, and \(n\) is the number of cartel members. One can see that \(\frac{f_1(c)}{1 - F_1(c)} = \frac{nf(c)}{1 - F(c)} > \frac{f_2(c)}{1 - F_2(c)} = \frac{f_2(c)}{1 - F_2(c)}\).

Studying the magnitude and determinants of umbrella damages requires the comparison of competitive equilibrium bid functions, i.e bid functions when the firms do not collude, with the collusive bid functions derived above. Denote by \(n\) the number of cartel members, or cartel size. For simplicity, assume that all bidders, whether or not in the cartel, are \(ex-ante\) symmetric, and that there is a single non-cartel firm. All firms draw their cost from a distribution \(F\) with support \([\xi, \bar{c}]\). \(F\) has a continuous density \(f\) strictly positive on \((\xi, \bar{c}]\). Under these assumptions, the competitive auction in which firms do not collude is a symmetric auction with \(n + 1\) bidders. There is a unique equilibrium in strictly increasing strategies (see Krishna (2006)).
**Assumption 1** (*Ex-ante* Symmetric IPV). *Across bidders, costs are symmetric (identically distributed according to the distribution $F$), independent, and private.*

Denote by $\beta$ the symmetric equilibrium bidding strategy, and by $\phi = \beta^{-1}$ the corresponding equilibrium inverse bid function. In this case, if a bidder’s cost is $c_i$, bidding $b$ yields expected profits given by:

$$\pi(b; c_i) = (b - c_i) \Pr(c_j > \phi(b), \forall j \neq i) = (b - c_i) (1 - F(\phi(b)))^n$$  \hfill (5)

The first-order condition with respect to $b$ (at $c_i = \phi(b)$) is:

$$\frac{nf(\phi(b))}{1 - F(\phi(b))} \phi'(b) = \frac{1}{b - c_i} \quad \text{with boundary condition } \phi(\bar{c}) = \bar{c}. \quad \text{ (6)}$$

Let $G(b) = F(\phi(b))$ denote the distribution of a firm’s equilibrium bid. Let $g(b)$ denote the corresponding density function. As bidders are symmetric, this distribution doesn’t depend on $i$. The first-order condition can be rewritten:

$$c_i = b - \frac{1 - G(b)}{ng(b)}$$ \hfill (7)

Equation (7) expresses the individual private cost $c_i$ as a function of the individual equilibrium bid $b$, and the distribution of equilibrium bid $G$. This mapping is at the core of the structural estimation of the cost distribution $F$ from the observed distribution of bids $G$ (see Guerre, Perrigne, and Vuong (2000) and step 2 in section 6.2).

### 2.1 Effect of collusion on the non-cartel firm’s profits

This subsection examines the effect of collusion between a strict subset of bidders on the bidding and profits of the non-colluding bidder—or, outsider. First, the outsider’s interim payoff (i.e., the expected profits conditional on his cost) always increases when his competitors form a cartel.

Define the outsider’s equilibrium interim payoff when facing $n$ competitors (no collusion):

$$\pi(c) = \max_b (b - c) [1 - F(\phi(b))]^n$$

Similarly, define the outsider equilibrium interim payoff when facing the cartel bidder (the $n$ competitors collude):

$$\tilde{\pi}(c) = \max_b (b - c) [1 - F_1(\phi_1(b))]$$
Assume first that the cartel mechanism is efficient, i.e. \( F_1(c) = 1 - (1 - F(c))^n \). Then we have the following lemma:

**Lemma 1.** If the cartel mechanism is efficient, the outsider’s interim payoff is strictly larger when his competitors collude:

\[
\tilde{\pi}(c) > \pi(c) \quad \text{for all } c \in [c, \bar{c}]
\]

**Proof.** Note that if \( \phi \) solve the symmetric \( n + 1 \) bidders procurement auction, it also solves the two bidder procurement auction in which bidders’ cost are drawn from \( 1 - (1 - F(c))^n \). When the \( n \) firms collude efficiently, we obtain a 2 bidders asymmetric auction in which bidders’ costs are drawn from \( F \) for the non-cartel firm and \( 1 - (1 - F(c))^n \) for the cartel bidder. Because \( F \) and \( 1 - (1 - F(c))^n \) can be ordered stochastically in the hazard rate sense, Corollary 1 of [Lebrun 1998](#) applies to obtain the strict inequality for the non-cartel firm profits.

In general, the cartel mechanism might not be efficient. Assume that the cartel bidder’s cost distribution \( F_1 \) (given a mechanism) satisfies the conditional stochastic dominance relation\(^{14}\)

\[
\frac{d}{dc} \left( \frac{1 - F_1(c)}{(1 - F(c))^n} \right) > 0 \quad \text{for all } c \in (c, \bar{c})
\]

**Proposition 2.** If (8) holds, the outsider’s interim payoff is strictly larger when his competitors collude:

\[
\tilde{\pi}(c) > \pi(c) \quad \text{for all } c \in [c, \bar{c}]
\]

**Proof.** \( F_1 \) and \( 1 - (1 - F(c))^n \) satisfy the assumption of conditional stochastic dominance, therefore Corollary 1 in [Lebrun 1998](#) implies that the outsider’s interim payoff is strictly larger when facing the cartel with the mechanism yielding \( F_1 \) than when facing an efficient cartel. Combining this observation with the previous lemma gives the result.

\[\square\]

### 2.2 Effect of the cartel size and cartel mechanism on the non-cartel firm’s bidding

In this subsection, comparative statics for the non-cartel firm equilibrium bid function are presented. In particular, two features are investigated: the cartel size and the cartel mechanism. Comparative statics in first-price asymmetric auctions have been studied by [Lebrun 1998](#). The results presented here are an application of the main theorem proved in the latter article to the specific case of a cartel bidding in a procurement auction.

\(^{14}\)Note that this relation can also be rewritten in terms of hazard rate dominance.
First, let the cartel mechanism be fixed. Using the same notations introduced above, denote by 1 the cartel bidder, and by 2 the non-cartel bidder \((F_i, \text{ for } i \in \{1, 2\})\) the corresponding cost distributions, which satisfy the assumptions of the model. The dependency of the cartel bidder’s cost distribution on the cartel size \(n\) is explicitly represented as: \(F_1(\cdot|n)\). In particular under Assumption 1 if the cartel is efficient: \(F_1(c|n) = 1 - (1 - F(c))^n\) (minimum cost among \(n\) symmetric bidders). If the cartel is inefficient: \(F_1(c|n) = F(c)\) (cartel bidder selected randomly).

Assume the cartel mechanism is such that \(F_1(\cdot|n)\) satisfies the following conditional stochastic dominance condition\(^1\)

\[
\frac{d}{dc} \left( \frac{1 - F_1(c|n)}{1 - F_1(c|n + 1)} \right) > 0 \quad \text{for all } c \in [\underline{c}, \overline{c}] \tag{9}
\]

**Proposition 3.** Assume that (9) holds. Let the bid functions and their inverses at the unique equilibrium when the cartel size is \(n\) be denoted \(\beta_1(\cdot|n), \beta_2(\cdot|n)\) and \(\phi_1(\cdot|n), \phi_2(\cdot|n)\) respectively. Then

1. As the cartel size increases, the non-cartel firm bids more aggressively:

   \[
   \beta_2(c|n) > \beta_2(c|n + 1) \quad \text{for all } c \in [\underline{c}, \overline{c}]
   \]

2. The larger cartel’s bid distribution is stochastically dominated by the smaller cartel’s bid distribution:

   \[
   F_1(\phi_1(b|n)|n) < F_1(\phi_1(b|n + 1)|n + 1) \quad \text{for all } c \in [\underline{b}, \overline{c}]
   \]

   where \(\underline{b} = \beta_1(\underline{c}|n) = \beta_2(\underline{c}|n)\)

3. As the cartel size increases, both the cartel and non-cartel bidders’ interim profits (conditional on their private cost) decrease

**Proof.** Follows directly from Theorem 1 and Corollary 1 in Lebrun (1998).

Condition (9) holds when the cartel mechanism is efficient. However, the condition doesn’t hold if the cartel mechanism is inefficient (\(F_1\) is independent of \(n\)).

Next, let the cartel size be fixed. The effect of the cartel mechanism on the non-cartel firm bidding can be analyzed. Consider two mechanisms implying a cartel bidder’s cost distribution of either \(F_1\) or \(\tilde{F}_1\). Assume the two distribution satisfy the conditional stochastic dominance relation\(^2\)

\[
\frac{d}{dc} \left( \frac{1 - F_1(c)}{1 - \tilde{F}_1(c)} \right) > 0 \quad \text{for all } c \in [\underline{c}, \overline{c}] \tag{10}
\]
This condition implies in particular that $\tilde{F}_1$ is first order stochastically dominated by $F_1$. For instance, if $F_1(c) = F(c)$ (inefficient cartel) and $\tilde{F}_1(c) = 1 - (1 - F(c))^n$ (efficient cartel), the condition holds.

**Proposition 4.** Assume (10) holds. Let the bid functions and their inverses at the unique equilibrium when the cartel bidder’s distribution is $F_1$ (resp. $\tilde{F}_1$) be denoted $(\beta_1, \beta_2)$ (resp. $(\tilde{\beta}_1, \tilde{\beta}_2)$) and $(\phi_1, \phi_2)$ (resp. $(\tilde{\phi}_1, \tilde{\phi}_2)$). Then

1. The non-cartel firm bids more aggressively when the cartel bidder’s cost distribution is $\tilde{F}_1$ than when it is $F_1$:
   $$\beta_2(c) > \tilde{\beta}_2(c) \quad \text{for all } c \in [c, \bar{c}]$$

2. The cartel’s equilibrium bid distribution can be ordered according to first-order stochastic dominance:
   $$F_1(\phi(b)) < \tilde{F}_1(\tilde{\phi}_1(b)) \quad \text{for all } c \in [b, \bar{c}]$$
   where $b = \beta_1(\bar{c}) = \beta_2(\bar{c})$

3. Cartel and non-cartel bidders’ interim profits (conditional on their private cost) are lower under $\tilde{F}_1$ than under $F_1$.

**Proof.** Follows directly from Theorem 1 and Corollary 1 in Lebrun (1998). □

The intuition for Proposition 3 and 4 is as follow. In both cases, as the cartel bidder is made stronger (either by increasing the size of the cartel or by making the cartel mechanism more efficient), the non-cartel firm responds by bidding more aggressively (part (1) of the propositions). As a consequence, the cartel bidder’s interim payoff decreases. In equilibrium, the cartel best response is such that it is more likely to bid lower: the new equilibrium bid distribution is first-order stochastically dominated by its initial equilibrium bid distribution (part (2) of the propositions). This results in the non-cartel firm interim payoff being lower as well.

### 3 The School Milk Market

Most public school districts in Texas use procurement auctions to allocate contracts for the supply of school milk. Every year, between May and August, each district sets a first-price sealed bid procurement auction, specifying contract characteristics such as the estimated quantities (in half pint) of milk to supply by milk categories, the number of delivery points, the contract period, the delivery times, and whether a cooler has to be provided.\footnote{The main categories of milk being: whole white, whole chocolate, low-fat white, low-fat chocolate.} Firms
have one month to prepare their bid, which is a price per half pint for each category of milk. A bid can be escalated or fixed. Escalated bids are indexed to the price of raw milk, to ensure the milk supplier against potentially large fluctuations of the price of raw milk over the contract period. Bidders have to sign a non-collusive affidavit, stating that they did not partake in any communication with other bidders regarding prices or participation and that they will not give or receive any side-payments. On the day of the letting, all submitted bids are opened, and the bidders’ identities and bids are publicly announced. Dairy distributors have to deliver packaged milk to various customers (e.g., retail stores, government agencies, and schools). Retail stores are the primary revenue source for distributors. School milk contracts typically form 10% to 20% of a distributors’ revenue.

By nature, the school milk market is remarkably exposed to collusion. Firms compete only on prices as the terms of the contract (quantity and quality) are fixed, and the product is homogenous. There are many small contracts to be gained facilitating market division. Bids and bidders’ identities are publicly announced, which helps to detect price cuts by cartel members and increases the transparency of prices. Firms frequently interact as auctions are not held on the same day, which permits retaliation in case of cheating. The demand for milk is inelastic, so price-increases will yield higher profits and are unlikely to face any buyer resistance. Finally, the market is relatively concentrated, helping coordination.

Some of the aforementioned market features also enable potentially large umbrella damages. Indeed, these damages stem from non-cartel firms adjusting their bidding behavior to the supra-competitive levels sustained by the cartel. This adjustment is feasible because all bids and bidders’ identities are publicly announced, which results in non-cartel firms learning the price level in the rigged districts and adapting to it. The latter channel is reinforced by the high frequency of interactions, due to the large number of contracts every year.

Next, the cost structure of milk processors is described. In anticipation of the structural analysis, it is useful to decompose a milk processor’s cost for a specific contract into:

1. A component common to all firms bidding for a contract: this component may depend on observed auction characteristics such as the quantity of milk to supply, whether bids can be escalated, whether coolers and straws have to be provided, the number of school within the district (which affects the quantity to supply and the number of delivery points), as well as the number of deliveries per week.

Additionally, this cost component depends on the price of raw milk, which is regulated by federal order, as well as processing, packaging and labor costs which, according to industry experts, are constant across firms in the market.\footnote{A federal milk order sets a uniform minimum price for raw milk in the area. This price is typically}
2. An idiosyncratic component specific to the milk processor: this cost first depends on the distance between the milk processing plant and the school district. More precisely, it depends on how close the school district is to the firm's distribution route. This distribution route depends on the firm's current portfolio of clients (e.g., government agencies, military bases, retail stores, and schools). Second, the idiosyncratic component depends on the firm current capacity utilization. If the firm is near capacity, winning a new contract may signify employing an additional truck and driver, which largely increases the cost of fulfilling the contract. Finally, idiosyncratic costs include a firm’s efficiency in packaging, loading trucks, and managing the machinery.

If the common component is controlled for, the cost structure falls in the independent and private value framework. Section 6.3.2 further discusses this assumption.

A description of the conspiracy, as well as the main actors in the industry, is provided next. In 1992 and 1993, nine milk processors accused of collusion in the DFW market area settled with the State. The cartel included the leading suppliers with plants in the DFW market area: Borden, Foremost, Schepps, Cabell, Oak Farms, Metzger, Vandervoort, Gandy, and Preston. Indictments suggest that collusion began at least as early as 1975. This article focuses on the period from 1980 to 1992. The cartel went through the following structural changes: in 1983, Borden acquired Metzger; in 1985, Preston entered the school milk market and joined the conspiracy; in 1986, Schepps acquired Foremost; in 1990, Cabell acquired Oak Farms.

Pure Milk Co. is the largest non-cartel school milk supplier in the dataset. The firm’s main plant was located in Waco, TX (i.e., in a different federal order zone than the DFW cartel). The company was founded in the 1960s and was successful in establishing a strong local customer base in Central Texas by marketing higher quality dairy products. Its main raw milk supplier was Dairy Farmers of America, a national milk marketing cooperative. School contracts made up to 20% of the firm’s business. Although Pure Milk bid primarily for contract near its plant in the Waco market area, it also participated in a non-negligible number of auctions for school districts in the DFW market area. According to the General Manager of the firm at that time, these were typically larger contracts justifying ”going the extra mile.” In these occasions, Pure Milk bid against the cartel. The article focuses on these contracts in which Pure Milk bid in districts where the cartel was operating. Pure Milk’s location further from the epicenter of the conspiracy may have played a non-negligible role increasing in the distance from the Midwest. In this article, the marketing area is the Texas Milk Marketing Order. Within the marketing area, prices differ by a fixed proportion from one zone to the other. The price of raw milk is around 7 cents per half-pint.

As in Pesendorfer (2000), collusion is thought of as an explicit or implicit scheme designed to limit competition and increase profits.
in it not being part of the cartel. Most of its business was conducted in the Waco market area, whereas the cartel was only active in Dallas-Fort Worth.

4 The Data

The article studies school-milk contracts awarded annually between 1980 and 1992 in three large market areas in Northeastern and Southern Texas: Dallas-Fort Worth, Waco and San Antonio. Contracts are awarded at the school district level. Figure I shows school districts in the dataset by market area. Each school district contains around four to five schools. Initially, the dataset contains information on 1,620 auctions, 4,444 bids.

The main dataset is the auction data. This dataset was collected by the Antitrust Division of the US Department of Justice during its investigation of the Texas milk cartel. For each contract awarded, the following characteristics are observed: the county and school district awarding the contract, the identity of the bidding firms and corresponding bids for each milk category (whole white, whole chocolate, low-fat white, low-fat chocolate, skim milk), the quantities required per milk category, whether a cooler has to be provided, the number of meals served in the school district, the school district enrollment, the number of deliveries per week, the number of schools in the district, whether a bid was fixed or escalated, and the identity of the winner.

Three auxiliary datasets complement the auction data. Two are obtained from the US Department of Agriculture’s Marketing Service. First, a dataset on prices of Class I fluid milk for the period of interest. This is the price of raw milk sold by milk cooperatives (such as Dairy Farmers of America) to milk processors and distribution firms. Second, a dataset giving the processing plant-locations of the firms bidding for the school milk contracts. Third, the longitudes and latitudes of school districts are added to the main auction data.

The reduced form and structural analysis are conducted on the data after the following preparation. Auctions with more than one winner are dropped. Auctions with only one participant are dropped. Prices are deflated using the CPI deflator into 1982 dollars. Finally, a distance variable is constructed for each observed bidder-auction pair: this variable measures the great-circle distance between the school district and the bidder’s closest plant. The variable is constructed using the latitude and longitude coordinates of firms’ plants and school districts. After this procedure, the dataset contains information on 1,033 auctions.

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18 In a few large and densely populated district, several distributors shared the contract.
19 Southwest Market Area (Federal Milk Order 126).
20 Class I Milk is raw milk destined to be used as a beverage, in opposition to milk processed into yogurts, cheese, etc.
21 Prices differ from one region to the other within the Texas Milk Marketing Order.
Table 1 shows that the majority of firms win between 20% and 30% of auctions in which they participate. Borden and Oak Farms are the largest firms by number of contracts won over this period. Pure Milk (the largest non-cartel firm) is in the second-tier of the distribution. It won 28% of the contracts on which it bid.

Table 2 gives summary statistics by market area. Over the period of interest, 735 contracts were awarded in Dallas-Fort Worth, 143 in San Antonio, and 179 in Waco. Only 15 school districts awarded contracts in San Antonio, against 30 for Waco. Indeed, school milk contracts in San Antonio are for larger quantities. The average winning bid (for a half-pint of whole white milk) is greater in Dallas-Fort Worth, followed by San Antonio, and lastly Waco. The average cost of a contract is the highest in San Antonio, reflecting again the larger size of contracts. The average contract cost in DFW is $85,115 against $18,779 in Waco. This cost difference reflects differences in contract sizes, raw milk prices, contract specifications across the two market areas but is also potentially related to inflated cartel prices in the DFW area.

5 Reduced Form Analysis

In this section, the bidding behavior of the largest non-cartel bidder (Pure Milk Co.) is examined. This preliminary analysis provides evidence that Pure Milk bid significantly less aggressively when facing cartel bidders and enables a preliminary assessment of the overbidding.

Pure Milk’s processing plant is located near the city of Waco, in McLennan county. The firm bid mainly in McLennan and neighboring counties (see Figure 1). However, in 10% of the cases, the firm bid in further counties, located in the cartel area of activity. Auctions in which Pure Milk bid are divided by counties into two separate types:

- Collusive auctions: these are in counties contiguous or close to Dallas-Fort Worth in which the cartel presence was established by the Department of Justice. The counties are Johnson, Hood, Erath, Dallas, and Comanche. Such auctions form around 10% of Pure Milk’s bids.

- Competitive auctions: these are in counties outside the Dallas-Fort Worth cartel territory (they are located in the Waco or San Antonio market areas). The counties are

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22 Both firms are national, with a larger distribution network.

23 Note that for these winning bid may reflect different auction characteristics from one market area to the other (quantities, input prices etc)
McLennan and directly contiguous counties: Coryell, Falls, Limestone, Bell, Hill, and Bosque.

This classification is based on the factual statement of the milk cartel prosecution. It is assumed that the cartel bid in the collusive counties, whereas all firms bid competitively in the competitive counties.

The logarithm of Pure Milk’s bid for whole white milk is regressed on the variables listed in Table 3. All continuous variables are in logarithm. Quadratic terms for the number of meals and distance are included. In the first specification, no fixed effects are used. In the second specification, year fixed effects and dummy variables for “collusive” and “competitive” counties are used. In the third specification, year fixed effects and county dummy variables are used. Results are shown in Table 4.

Table 4 shows that bids move closely with the price of raw milk. As expected, escalated bids are lower than fixed bids (because they are indexed to the price of raw milk and therefore shield bidders against future fluctuations of their input price). Bids are increasing in the number of schools to supply within the district. Bids are decreasing in the number of bidders. In specifications (2) and (3), bids are increasing and convex in the number of meals served (equivalently in the quantity of milk to supply). This pattern would be consistent with an optimal utilization rate for milk distributors. In specifications (2) and (3), bids are concave in the distance between the processing plant and the school district. Distance increases the cost of fulfilling a contract, but with diminishing effects.

The regression provides evidence that Pure Milk overbid when facing the cartel. In specification (2), the coefficient on the dummy for the group of collusive auctions defined above is significantly different from zero and positive. Pure Milk bid on average 6% higher in the collusive auctions (facing the cartel) relative to the competitive auctions. Specification (3) breaks down the effect at the county level. As shown in Figure 2, coefficients are significantly positive in collusive counties, whereas the coefficients are significantly negative in counties with competitive auctions. All coefficients are measured with respect to the average bid in Bell county (competitive auction).

The reduced form analysis demonstrates that the largest non-cartel bidder overbid when facing the cartel. This approach is, however, limited if one is interested in the magnitude of umbrella damages relative to cartel damages, or in assessing the size of inefficiencies introduced by the cartel agreement. Therefore, more structure is imposed on the data. This structure is derived from the theoretical model of section 2. The structural analysis allows

24The log-log specification allows the interpretation of coefficients as elasticities. Additionally, because bids are positive, errors are positively skewed. By logging the observed variable, errors are made more symmetric.
not only the estimation of damages and inefficiencies, but also a finer decomposition of umbrella damages into damages stemming directly for the non-cartel firm overbidding, and damages originating in the inefficiency of the asymmetric auction.

6 Structural Analysis

In this section, the theoretical model of section 2 is estimated from the data. The objectives are to quantify, first, the average damages caused to the school districts by the outsider firm (i.e., umbrella damages), and second, the loss in efficiency due to asymmetries between bidders introduced by the cartel agreement.

The estimation approach begins by recovering the underlying cost distribution of bidders from observed competitive bids. As auctions differ in their specifications (in the quantity to be supplied, whether bids can be escalated, number of deliveries per week, number of schools in the district), bids will reflect auction specific heterogeneity. A first step is to account for this observed auction heterogeneity to obtain a set normalized bids. Costs are then estimated non-parametrically using the empirical distribution of normalized bids following Guerre, Perrigne, and Vuong (2000)—GPV hereafter.

Using the estimated cost distribution, I simulate counterfactual bids by solving the auction in which the outsider firm faces the cartel. Two scenarios are considered: (1) assuming that the cartel mechanism is efficient (in the sense, that the cartel member with the lowest cost bids on behalf of the cartel), or (2) assuming that the cartel mechanism is inefficient (the cartel bidder is selected randomly from the cartel members). The cartel bidder is ”stronger” when the mechanism is efficient (scenario (a)) and therefore, the outsider will tend to bid more aggressively (see section 2). As a result, estimates under this scenario will give a lower bound on umbrella damages. Estimates under scenario (b) will accordingly provide an upper bound on umbrella damages.

Under scenario (a), cartel bidder and outsider draw their costs from different distributions. Equilibrium bid function in such asymmetric auction cannot be solved for analytically. The bid functions are obtained by numerical resolution of the system of differential equations (3). The method used is the fixed point iteration, introduced by Fibich and Gavish (2011).

Once the counterfactual bids are obtained, auction heterogeneity is added back to the bids, to reflect characteristics of auctions in which the cartel bid against the outsider firm. Damages to the seller (school district) and inefficiencies are estimated.
6.1 Data limitations

Although the dataset used is rich in many dimensions, a few difficulties must be dealt with before laying out the estimation approach. A first issue is due to auctions differing in their observed characteristics, such as the quantity of milk to supply. Auction observed heterogeneity is addressed by imposing additional structure on firms’ costs (Assumption 2). A second issue is related to the relatively small sample size of auctions with a non-cartel firm bidding against the cartel. This issue is compounded by the fact that the cartel mechanism is unknown. This section shows how assumptions restricting the structure of the model can be leveraged to estimate the variables of interest (cost distribution, equilibrium bid functions, and damages).

Denote by $x_d$, auction $d$’s observed characteristics (such as the quantity to be supplied, whether bids can be escalated, number of deliveries per week, number of schools in the district, etc.).

Assumption 2 (Multiplicative Separability). Bidder $i$’s cost in auction $d$, denoted $c_{id}$, can be written:

$$c_{id} = \tilde{c}_{id} \Gamma(x_d)$$

for some function $\Gamma(\cdot)$. $\tilde{c}_{id}$ represents bidder $i$’s idiosyncratic ("normalized") cost in auction $d$, which is independent from $x_d$.

Assumption 3 (Independence of costs across auctions). Costs are drawn independently across school districts and years.

A detailed discussion of these assumptions’ validity is delayed to Section 6.3.2.

The main issue is related to the estimation of the cost distribution from observed bids. The structural model developed in Section 2 requires (at least) two participants bidding non-cooperatively in each auction. Based on the list of firms prosecuted by the DoJ, all firms located in the Dallas-Fort Worth area were found colluding for contracts in that area. As a consequence, it is safe to assume that the majority of bids for auctions in the Dallas-Fort Worth area (in which all participants were cartel members) were either (1) a winning bid submitted by the cartel bidder to match the seller’s (underlying) reserve price or (2) cooperative "phony" or "complementary" bids submitted by other cartel bidders. The structural model cannot be used to infer underlying costs from these complementary bids (and reserve prices), as most of them do not necessarily map to a firm’s true cost, but were merely designed to simulate competition among bidders (or extract all of the seller’s surplus). Auctions in which the cartel participates are referred to as "collusive" auctions.

25The data contains 250 bids by the non-cartel firm, 33 of which are against the cartel
As the goal is to quantify the size of umbrella damages as a fraction of cartel damages, a natural way to solve the previous issue would be to use only the subset of collusive auctions in which the outsider firm bid against the cartel. In these auctions, at least two participants (the outsider and the cartel bidder) are bidding non-cooperatively. Unfortunately, two reasons make this alternative unappealing. First, the small sample size of this set of auctions (with the outsider bidding against the cartel) renders any non-parametric estimation of the distribution of costs impossible. Second, the cartel mechanism (in particular the choice of the cartel bidder) is a priori unknown. One could assume a mechanism for the selection of the cartel bidder, and estimate underlying costs from observed bids. However, such an assumption will be hard to justify. Additionally, if the mechanism is misspecified, this approach will lead to biased estimates of underlying costs. A more conservative approach is therefore preferred.

In the preferred approach, auctions from the two other market areas in the dataset (Waco and San Antonio) are used. As no firm was prosecuted in counties around Waco and San Antonio, I assume that firms were bidding competitively in these counties (denoted hereafter "competitive" auctions). By Assumption 2, observed auction heterogeneity can be separated from the idiosyncratic part of the bid. By Assumption 3, the set of competitive "normalized" bids can be used to recover the distribution of "normalized" costs (the $\tilde{c}_{id}$). Using the estimated distribution of costs, I simulate counterfactual bids by solving the asymmetric auctions in which the outsider firm bid against the cartel, for different specifications of the cartel mechanism (efficient cartel or inefficient cartel). Finally, by Assumption 2 (in particular the independence of "normalized" bids/costs and observed auction characteristics), observed auction characteristics drawn from the set of collusive auctions with the outsider firm bidding against the cartel, are added back into the bids. By following this procedure, I obtain a set of competitive auctions, and a set of corresponding collusive auctions with the outsider bidding against the cartel.

Advantages of this approach are twofold. First, estimation of the cost distribution does not hinge on the correct specification of the cartel mechanism. Indeed, costs are obtained from competitive auctions in which no bidders collude. Second, the cost distribution is estimated non parametrically, as the sample size of competitive auctions allows it.

One drawback of this approach is that it does not recover the specific cost corresponding to observed bids for collusive auctions in which the outsider participates. Instead, it computes upper and lower bounds on umbrella damages using (1) the cost distribution estimated from

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26 The outsider firm bid against the cartel in 33 auctions.
27 Pesendorfer (2000) finds evidence that the Texas cartel was quasi-efficient even though side-payments were not used, by comparing it to the Florida cartel which was efficient.
the set of competitive auctions and (2) auction observed characteristics drawn from collusive auctions in which the outsider participates.

### 6.2 Estimation Approach

The steps followed in the estimation are described below:

- **Step 1: Observed auction heterogeneity**
  The estimation procedure assumes that the data available is from auctions of *ex-ante* identical contracts. This assumption is not valid for school milk contracts. Indeed, contracts differ in various dimensions, which are public information and observed by the bidders before submitting their bids. This public information will enter not only a bidder’s private cost of realizing the contract but also his belief about other bidder’s costs.

  Haile, Hong, and Shum (2006) propose one method to account for auction-specific observed heterogeneity. The article shows that multiplicative separability (Assumption 2) of idiosyncratic costs and auction characteristics carries out to bids and auction characteristics.

**Lemma 2.** Assume the multiplicative separable structure:

\[
c_{idt} = \tilde{c}_{idt}\Gamma(x_{dt})
\]

where \(c_{idt}\) is bidder \(i\)'s cost for contract \(d\) at time \(t\), \(x_{dt}\) are contract-time specific characteristics. Then the equilibrium bid function has the multiplicative separable form:

\[
\beta(c_{idt}) = \beta(\tilde{c}_{idt})\Gamma(x_{dt})
\]

This result can be used to account for observed auction heterogeneity and homogenize the bids. Assume the following parametric specification: \(\Gamma(x_{dt}) = \exp(x'_{dt}\beta)\). The first-stage regression is:

\[
\ln b_{idt} = x'_{dt}\beta + \eta_t + \gamma_c + \kappa_i + n_{dt}\delta + \sigma_{idt}
\] (11)

where \(b_{idt}\) denotes the bid of bidder \(i\) for contract \(d\) at time \(t\), \(\eta_t\) is a time-specific dummy, \(\gamma_c\) is a county dummy, \(\kappa_i\) is a bidder-specific dummy, and \(n_{dt}\) is the number of bidders participating in contract \(d\) at time \(t\), \(\sigma_{idt}\) is the error term. \(x_{dt}\) include variables for the price of raw milk, the number of meals served (and its square), whether bids
can be escalated, whether a cooler has to be provided, the number of deliveries per week, and the number of schools in the school district. All continuous variables are in logarithm. Note that the use of time dummies enables us to capture seasonality: for instance, common packaging, processing, and labor costs, that might change over time but are common to all bidders.

The first-stage regression is run on the sample of competitive auctions, that is, auctions in which the cartel did not participate.

Normalized bids are constructed from the results of regression \( \ln \tilde{b}_{idt} = \ln b_{idt} - x'_{idt}\hat{\beta} - \hat{\eta}_t - \hat{\gamma}_c \).

As equilibrium bid functions depend on the number of bidders participating in the auction, the rest of the estimation is conducted on auctions fixing the number of participants to three bidders.\(^{28}\)

The estimation approach abstracts from auction unobserved heterogeneity. The latter would be relevant if bidders were to observe auction characteristics that are unobserved by the econometrician. In the context of school milk auctions, there do not seem to be other factors relevant to firms’ costs aside from the ones controlled for in this first estimation step, i.e., input prices, quantities, and auction specifications (escalated bids, coolers, number of deliveries, etc.).

- **Step 2: Estimation of the underlying cost distribution (GPV estimator)**
  Following the procedure presented in Guerre, Perrigne, and Vuong (2000), the underlying distribution of costs can be estimated using the distribution of normalized bids, obtained in the previous step. The cumulative distribution of bids is estimated using the empirical distribution function, and the density is estimated using a kernel with finite support.\(^{29}\) Bid data are trimmed in order to control for the asymptotic bias at the boundaries of the support of the bid distribution, as suggested in Guerre, Perrigne, and Vuong (2000).

- **Step 3: Derivation of the asymmetric equilibrium bid functions**
  As shown in Section 2, the cartel bidder and non-cartel firm equilibrium (inverse) bid functions, denoted \( \phi_1 \) and \( \phi_2 \) respectively, are the solutions of:

\[
\begin{align*}
\frac{1}{b - \phi_2(b)} &= \frac{f_1(\phi_1(b))\phi_1'(b)}{1-F_1(\phi_1(b))} \\
\frac{1}{b - \phi_1(b)} &= \frac{f_2(\phi_2(b))\phi_2'(b)}{1-F_2(\phi_2(b))}
\end{align*}
\]

\(^{28}\)Note that auctions with two participants do not give rise to potential umbrella damages, whereas auctions with more than four bidders are somewhat scarce in the dataset.

\(^{29}\)The epanechnikov kernel is used. The bandwidth selection method is likelihood cross-validation.
where $F_i$ (resp. $f_i$) is the cumulative distribution (resp. density function) of costs of the cartel ($i = 1$) and the outsider firm ($i = 2$). Along with the boundary conditions $\phi_1(b) = \phi_2(b) = c$ (lower bound of support of bids) and $\phi_1(\bar{c}) = \phi_2(\bar{c}) = \bar{c}$, equation 12 forms a nonlinear boundary value problem (BVP), in which the location of the left-boundary $b$ is unknown. In addition to this latter ”non-standard” feature, the numerical resolution of the BVP is furthermore complicated (see below) by the fact that the mapping $M$ in $(\phi_1'(b), \phi_2'(b)) = M(b, \phi_1(b), \phi_2(b))$ is not Lipschitz-continuous in $(\phi_1(b), \phi_2(b))$ at the right boundary $\bar{c}$.

The BVP defined in equation 12 cannot be solved analytically. However, several numerical solutions have been proposed in the literature: the backward-shooting method was first used to solve this problem by Marshall, Meurer, et al. (1994) and used and refined in various subsequent articles. Although this method is currently the standard for computing equilibrium bids in asymmetric auctions, it suffers from large instability at the right boundary (see Fibich and Gavish (2011) for a detailed analysis). As a consequence, an alternative numerical method, proposed in the latter article, is preferred. Their idea is to recast the BVP as a system of differential equations in $\phi_1$ and $b$ (instead of $\phi_2$) as functions of $\phi_2$. This allows me to transform the BVP with unknown left boundary into a BVP with known boundaries, and apply standard numerical techniques such as fixed-point iteration on a grid. The details of their numerical method, adapted to this setting, are presented in Appendix A.

As the internal organization of the cartel is unknown (in particular regarding how the cartel bidder is selected for a given contract), two extreme cases for the cartel internal mechanism are considered. These two scenarios will provide upper and lower bounds on umbrella damages.

1. Efficient cartel mechanism: the cartel is able to select its lowest cost member to bid on behalf of the cartel at the auction. Such a mechanism can be sustained, for instance, if the cartel uses side-payments. Denote by $F$ the true distribution of costs for each bidder (assuming symmetry) and by $n$ the number of cartel members submitting bids in the auction. Then the cost distributions of the non-cartel firm and the cartel bidder are:

$$F_2(c) = F(c) \quad f_2(c) = f(c) \quad \forall c \in [c, \bar{c}]$$

\[ F_1(c) = 1 - (1 - F(c))^n \quad f_1(c) = n f(c)(1 - F(c))^{n-1} \quad \forall c \in [c, \bar{c}] \]

Note that \( F_1 \) is simply the distribution of the minimum of \( n \) random variables drawn independently from \( F \).

2. Inefficient cartel mechanism: for each contract, the cartel bidder is selected randomly among cartel members. In this case:

\[ F_2(c) = F_1(c) = F(c) \quad f_2(c) = f_1(c) = f(c) \quad \forall c \in [c, \bar{c}] \]

Estimators of \((F_1, F_2, f_1, f_2)\) are constructed from the non-parametric estimators of \((F, f)\) obtained in Step 2 (GPV estimation) and passed on to the fixed point iteration algorithm. The result of the numerical method are estimators of the true asymmetric equilibrium inverse bid functions: \( \hat{\phi}_1 \) and \( \hat{\phi}_2 \).

- **Step 4: Reincorporation of the observed auction heterogeneity**
  Observed auction heterogeneity is added back into the set of normalized bids by drawing from the empirical distribution of auction characteristics in cases where the outsider firm bid against the cartel. This is motivated by Assumptions 1 and 2.

- **Step 5: Damage Assessment**
  Estimates of damages are constructed by combining the set of competitive winning bids and corresponding counterfactual set of collusive winning bids. Estimates of efficiency losses are constructed by comparing the winner’s cost in the competitive and corresponding collusive auction.

### 6.3 Results

#### 6.3.1 Assessment of Damages and Inefficiencies

Figure 3 shows the estimated bid functions in the case of three bidders auctions. Each panel shows the bid function in the competitive auctions, along with the counterfactual outsider and cartel’s bid functions in the case of an efficient cartel mechanism (left panel), and an inefficient cartel mechanism (right panel). In both cases, the collusive bidding functions lie above the competitive bidding function as both the cartel bidder and the outsider bid

\[ ^{31} \text{The estimator of the cumulative distribution } F \text{ is kernel-smoothed in order to facilitate the numerical resolution.} \]

\[ ^{32} \text{Collusive auctions are the auctions in which the outsider firm bid against the cartel.} \]

\[ ^{33} \text{Normalized bid functions are represented.} \]
less aggressively. Note that the overbidding increases the smaller is the bidder’s cost. Over-
 bidding when the cartel is inefficient is larger than when the cartel is efficient (see section 2).

Table 5 presents damage estimates obtained from the structural analysis detailed in section 6.2. Estimates of damages to the auctioneer (school district) correspond to the difference between the winning bid of the collusive auction in which the outsider firm bids against the cartel and the winning bid of the competitive auction. Damages are classified into three types, depending on the identity of the winner in each of the competitive and collusive auction:

1. **Cartel damages**: these are damages in collusive auctions won by the cartel. From the estimated bid function, the cartel also wins the competitive auction. These are the typical damages antitrust authorities try to assess.

2. **Misallocation damages**: these are damages in collusive auctions won by the outsider when a cartel member would have won the competitive auction. When the cartel mechanism is efficient, it introduces asymmetry in the auction. As a result, the outcome of the auction is no longer efficient. The outsider wins the collusive auction even if they are not the lowest cost bidder. The reason is that the cartel bids less aggressively than the outsider.

3. **Outsider damages**: these are damages in collusive auctions won by the outsider when the outsider would have won the competitive auction as well. The outsider is the lowest cost bidder. In this situation, damages to the auctioneer come merely from the fact that, by best replying to the cartel’s bidding, the outsider can bid less aggressively than in the competitive auction.

Importantly, these damages are computed conditional on the cartel (in the case of cartel damages) or the non-cartel firm (for misallocation and outsider damages) winning the collusive auction. When the cartel is efficient, auctions leading to cartel damages form 58% of the sample, auctions leading to misallocation damages form 9% of the sample, whereas auctions leading to outsider damages form 33% of the sample. When the cartel is inefficient, auctions leading to cartel damages form 50% of the sample, whereas auctions leading to outsider damages form 50% of the sample. Upper bounds (UB) and lower bounds (LB) are derived for outsider damages in the case of inefficient and efficient cartel mechanisms, respectively.

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34 The cartel bids less aggressively in the collusive auction, so if the cartel wins, it must be the lowest cost bidder.

35 The situation in which the outsider wins the competitive auction and the cartel wins the collusive auction does not arise because the outsider bids more aggressively than the cartel.
In auctions where the outsider firm bid against the cartel, the mean damages per contract in the whole white milk category are between $1,052 and $2,019. If the same overcharge were applied to all milk categories, damages per contract would be between $3,906 and $7,495. For auctions won by the outsider as the lowest cost bidder (outsider damages), the winning bid is 2.9% to 8.5% above the competitive winning bid, depending on the cartel mechanism. As a fraction of the winning competitive mark-up, these damages are between 12% and 42%. For auctions won by the outsider, while the cartel was the lowest cost bidder, misallocation damages are 3.5% of the competitive winning bid or 26% of the competitive winning mark-up. For auctions won by the cartel, damages are between 5.9% and 8.4% of the competitive winning bid, or between 30% and 42% of the competitive winning mark-up.

Inefficiencies introduced by the (efficient) cartel agreement are measured by the difference in the winner’s cost in the competitive and collusive auctions. These two costs differ only in the case of misallocation damages, when the outsider firm wins the collusive auction, while the cartel would have won the competitive auction. Inefficiencies amount to an increase of 3.7% of the winner’s cost, equaling 788$ per contract in the whole white category, or 2,909$ per contract.\(^{36}\)

Damages caused by the outsider’s bidding behavior form a non-negligible fraction of cartel damages. These umbrella damages can be decomposed into outsider damages and misallocation damages, as defined above. Per contract, outsider damages (conditional on the outsider winning) are estimated to be at least 47% of cartel damages (conditional on the cartel winning). In other words, the ratio of expected damages to the auctioneer conditional on the non-cartel firm winning over the expected damages to the auctioneer conditional on the cartel firm winning is estimated to be at least 47%. This lower bound is obtained with an efficient cartel. If the cartel is inefficient, outsider damages are as large as cartel damages. Misallocation damages are estimated to be as large as 64% when the cartel is efficient, and the auction is asymmetric. The estimates found for outsider damages, between 2.9% and 8.5% of the competitive winning bid, are consistent with the overcharge of 6% estimated in the reduced form section.

6.3.2 Robustness of the Assumptions underlying the Estimation Approach

This section discusses the validity of the assumptions imposed so far. The model is cast within the symmetric IPV framework (Assumption 1). A bidder’s cost can be decomposed into (1) a component common to all bidders, which includes not only observed auction

\(^{36}\)If the cartel is inefficient, misallocation damages might arise. However, they are due to the random selection of the cartel bidder rather than asymmetries between bidders. If the competitive auction would have been won by a cartel member that is not selected as the cartel bidder, there will be a loss of efficiency. This type of misallocation is left aside as it is not stemming from asymmetries.
characteristics but also common processing, packaging, and labor costs (2) a bidder-specific component.\footnote{See the discussion in \cite{PorterZona1999}.}

The symmetric IPV assumption is imposed on the bidder-specific component of costs (i.e., once the common component of costs has been filtered out). A bidder’s idiosyncratic cost depends on how close the school district is to its current delivery route. This delivery route depends on the bidder’s current portfolio of clients (which include government agencies, hospitals, military bases, etc.). In the same line, a bidder’s idiosyncratic cost depends on its current capacity utilization. Finally, idiosyncratic costs depend on the bidder’s efficiency in packaging, loading trucks, managing the machinery. Such factors are private to each bidder, as they do not affect its competitors’ costs. Moreover, these factors are independent across bidders.

Firms could, in principle, monitor each other’s capacities and portfolio of clients, and therefore derive information about how far a competitor’s route is from the school district of interest. Such public information would create ex-ante asymmetries across bidders: the cost distribution of a bidder depends on its plant-school district distance. Nonetheless, the symmetry assumption is imposed for two reasons: first, in the vast majority of cases, firms favor closer school districts; second, according to industry professionals, information about competitors’ distance from the school district of interest is more important to infer competitors’ participation decision rather than their actual cost realization.

In order to conduct the structural estimation of the cost distribution, independence of idiosyncratic costs across school districts and years is needed (Assumption 3). The use of time fixed-effects in the first step of the estimation approach helps capture any within-year correlation between costs drawn. The use of county dummies controls for within-county correlation of costs.

Another crucial assumption for the structural analysis is that the equilibrium derived is the one played in the data (i.e., in the competitive auctions). In particular, bidders know the number of firms participating in the auction. According to industry professionals, the number and the identity of the firms who are likely to bid for a given contract are usually known in advance: firms bid for the same set of contracts every year and therefore develop a good understanding of which competitors will be interested in a given contract. Information about which competitors are the closest to the school district of interest also helps in refining their estimate of the number of bidders.

The last issue to deal with concerns endogenous selection in the case of the outsider. Indeed, it is assumed that the cost distribution of the outsider is the same in the competitive and collusive auctions. This need not be the case if there is selection on observable auction
characteristics (i.e., the characteristics of the competitive and collusive auctions in which the outsider participates are systematically different) or on idiosyncratic costs (for instance if the outsider bids against the cartel only when the cost drawn is favorable enough). The first type of selection is dealt with by the separability assumption. Indeed, damages are estimated by drawing from the empirical distribution of collusive auction characteristics (cf step 4). The second type of selection, on idiosyncratic costs, seems less of a concern: according to the interviewed non-cartel bidder, the main factor driving participation in collusive auctions was the size of the contract.

7 Conclusion

This article examines how non-cartel firms’ bidding behavior can be affected by the existence of a cartel in a first-price procurement setting. In the case of the Texas school milk cartel, the analysis shows that the largest non-cartel firm bid significantly higher when facing the cartel than under competitive conditions. The structural model shows that conditional on the non-cartel firm winning against the cartel, damages to the auctioneer (in the form of inflated winning bids) are a non-negligible fraction of the damages caused when the cartel wins.

These results provide new evidence on the potential severity of umbrella damages and their ability to broaden the scope of overall damages caused by a cartel. The recent decision by the ECJ to allow “umbrella claimants” to pursue treble damages against cartels seems to recognize the latter fact, albeit pointing at the same time to the difficulty of proving such claims.

Some open questions remain. First, the article focuses on the school milk industry in which contracts are awarded via first-price procurement auctions. However, umbrella damages might not be restricted to auction environments. Alternative environments include industries where firms compete in quantities (an example would be the vitamin market and other similar commodities markets). Investigating the prevalence of such damages in alternative environments is left for future research. Second, in auction environments, the framework presented does not address the potential effect of a cartel existence on outsiders’ participation decision. If entry is endogenous and selective, a cartel will induce more entry by outsiders (relative to a competitive environment with the same number of potential bidders). Indeed, conditional on entry, outsiders’ profits are strictly larger when their competitors collude (see section 1.1). This raises interesting questions, which could be tested in setting with costly participation.
References


### Tables

#### Table 1: Number of bids and wins by firm 1980 – 1992

<table>
<thead>
<tr>
<th>Vendor</th>
<th># of bids</th>
<th># of wins</th>
<th>% of wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>BORDEN</td>
<td>836</td>
<td>232</td>
<td>0.278</td>
</tr>
<tr>
<td>CABELL</td>
<td>418</td>
<td>140</td>
<td>0.335</td>
</tr>
<tr>
<td>FOREMOST</td>
<td>139</td>
<td>45</td>
<td>0.324</td>
</tr>
<tr>
<td>GANDY</td>
<td>23</td>
<td>5</td>
<td>0.217</td>
</tr>
<tr>
<td>KNOWLTON</td>
<td>12</td>
<td>1</td>
<td>0.083</td>
</tr>
<tr>
<td>LILLY</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>METZGER</td>
<td>21</td>
<td>4</td>
<td>0.190</td>
</tr>
<tr>
<td>OAK FARMS</td>
<td>530</td>
<td>184</td>
<td>0.347</td>
</tr>
<tr>
<td>PRESTON</td>
<td>333</td>
<td>98</td>
<td>0.294</td>
</tr>
<tr>
<td>PURE</td>
<td>255</td>
<td>72</td>
<td>0.282</td>
</tr>
<tr>
<td>SCHEPPS</td>
<td>528</td>
<td>115</td>
<td>0.218</td>
</tr>
<tr>
<td>SUPERIOR</td>
<td>46</td>
<td>30</td>
<td>0.652</td>
</tr>
<tr>
<td>VANDERVOORT</td>
<td>340</td>
<td>107</td>
<td>0.315</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3,488</strong></td>
<td><strong>1,033</strong></td>
<td></td>
</tr>
</tbody>
</table>

#### Table 2: Descriptive Statistics by Market Area

<table>
<thead>
<tr>
<th>Market Area</th>
<th>DFW</th>
<th>San Antonio</th>
<th>Waco</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bids</td>
<td>2411</td>
<td>555</td>
<td>591</td>
</tr>
<tr>
<td>Number of contracts</td>
<td>735</td>
<td>143</td>
<td>179</td>
</tr>
<tr>
<td>Number of contracts/year</td>
<td>56.5</td>
<td>11</td>
<td>13.7</td>
</tr>
<tr>
<td>Number of counties</td>
<td>25</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Number of school districts</td>
<td>115</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>winning bid/half pint</td>
<td>Mean 0.1442</td>
<td>0.1417</td>
<td>0.1365</td>
</tr>
<tr>
<td></td>
<td>SD 0.0214</td>
<td>0.0283</td>
<td>0.0226</td>
</tr>
<tr>
<td># of meals per year-school district</td>
<td>Mean 522,578.2</td>
<td>1,273,001.4</td>
<td>339,188.0</td>
</tr>
<tr>
<td></td>
<td>SD 655,877.1</td>
<td>1,436,814.3</td>
<td>620,308.4</td>
</tr>
<tr>
<td>Contract total cost per school district</td>
<td>Mean 85,115.37</td>
<td>168,938.34</td>
<td>18,779.90</td>
</tr>
<tr>
<td></td>
<td>SD 112,916.49</td>
<td>202,501.19</td>
<td>24,883.69</td>
</tr>
</tbody>
</table>

*Note: Bids are for a half-pint of whole white milk. Bids and contract cost are in 1982 dollars.*
Table 3: List of variables

<table>
<thead>
<tr>
<th>Type</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auction specific</td>
<td>FMO price</td>
<td>raw milk price in 1982 dollars</td>
</tr>
<tr>
<td></td>
<td>meals</td>
<td>number of school lunches per school district per year</td>
</tr>
<tr>
<td></td>
<td>escalated</td>
<td>equals 1 if the bid is escalated</td>
</tr>
<tr>
<td></td>
<td>deliveries</td>
<td>number of deliveries per week</td>
</tr>
<tr>
<td></td>
<td>number of schools</td>
<td>number of schools in the school district</td>
</tr>
<tr>
<td></td>
<td>cooler</td>
<td>equals 1 if a cooler needs to be provided</td>
</tr>
<tr>
<td></td>
<td>number of bids</td>
<td>number of bids submitted in the auction</td>
</tr>
<tr>
<td>Bidder specific</td>
<td>distance</td>
<td>great-circle distance between closest plant and school district</td>
</tr>
<tr>
<td></td>
<td>bid</td>
<td>bid submitted for whole white category</td>
</tr>
<tr>
<td></td>
<td>incumbency</td>
<td>equals 1 if bidder won the contract in previous year</td>
</tr>
</tbody>
</table>

*Note: Indicators for missing values of cooler, deliveries and escalated are also included*
Table 4: Determinants of Pure Milk’s bids

<table>
<thead>
<tr>
<th>Dependent Variable: Whole white bid (in logarithm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Incumbency</td>
</tr>
<tr>
<td>FMO price</td>
</tr>
<tr>
<td>Meals</td>
</tr>
<tr>
<td>Meals sqrd</td>
</tr>
<tr>
<td>Escalated</td>
</tr>
<tr>
<td>Escalated missing</td>
</tr>
<tr>
<td>Cooler</td>
</tr>
<tr>
<td>Cooler missing</td>
</tr>
<tr>
<td>Deliveries</td>
</tr>
<tr>
<td>Deliveries missing</td>
</tr>
<tr>
<td>Number of schools</td>
</tr>
<tr>
<td>Number of bids</td>
</tr>
<tr>
<td>Distance</td>
</tr>
<tr>
<td>Distance sqrd</td>
</tr>
<tr>
<td>Collusive auction</td>
</tr>
<tr>
<td>COMAL</td>
</tr>
<tr>
<td>BOSQUE</td>
</tr>
<tr>
<td>COMAL</td>
</tr>
<tr>
<td>COMANCHE</td>
</tr>
<tr>
<td>CORYELL</td>
</tr>
<tr>
<td>DALLAS</td>
</tr>
<tr>
<td>ERATH</td>
</tr>
<tr>
<td>FALLS</td>
</tr>
<tr>
<td>HILL</td>
</tr>
<tr>
<td>HOOD</td>
</tr>
<tr>
<td>JOHNSON</td>
</tr>
<tr>
<td>LIMESTONE</td>
</tr>
<tr>
<td>McLLENNAN</td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>Year FE</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R²</td>
</tr>
<tr>
<td>Adjusted R²</td>
</tr>
<tr>
<td>Residual Std. Error</td>
</tr>
<tr>
<td>F Statistic</td>
</tr>
</tbody>
</table>

Note: A dummy for competitive auctions is omitted in specification (2). Omitted county is BELL in specification (3). Counties corresponding to collusive auctions in bold in specification (3). All continuous variables in log. Prices in 1982 $. ***Significant at the 1 percent level. **Significant at the 5 percent level. *Significant at the 10 percent level.
Table 5: Estimates of damages

<table>
<thead>
<tr>
<th></th>
<th>Point Estimate</th>
<th>LB of 90% CI</th>
<th>UB of 90% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean damage per half-pint ($) - LB</td>
<td>0.00518</td>
<td>0.00447</td>
<td>0.00579</td>
</tr>
<tr>
<td>Mean damage per half-pint ($) - UB</td>
<td>0.00982</td>
<td>0.00890</td>
<td>0.01069</td>
</tr>
<tr>
<td>Mean damage per contract (whole white only) ($) - LB</td>
<td>1,052.26</td>
<td>852.62</td>
<td>1,264.88</td>
</tr>
<tr>
<td>Mean damage per contract (whole white only) ($) - UB</td>
<td>2,019.22</td>
<td>1,663.09</td>
<td>2,395.20</td>
</tr>
<tr>
<td>Mean damage per contract ($) - LB</td>
<td>3,906.65</td>
<td>3,068.04</td>
<td>4,765.71</td>
</tr>
<tr>
<td>Mean damage per contract ($) - UB</td>
<td>7,495.49</td>
<td>5,966.52</td>
<td>9,097.94</td>
</tr>
</tbody>
</table>

| Mean damage per half-pint ($) |
| Outsider damages - LB         | 0.00316        | 0.00246      | 0.00379      |
| Outsider damages - UB         | 0.00919        | 0.00815      | 0.01019      |
| Misallocation damages         | 0.00404        | 0.00294      | 0.00516      |
| Cartel damages1               | 0.00647        | 0.00577      | 0.00713      |

| Mean damage as fraction of competitive winning bid |
| Outsider damages - LB         | 0.02996        | 0.02306      | 0.03648      |
| Outsider damages - UB         | 0.08538        | 0.07443      | 0.09614      |
| Misallocation damages         | 0.03538        | 0.02541      | 0.04637      |
| Cartel damages                | 0.05968        | 0.05279      | 0.06631      |

| Mean damage as fraction of competitive winning mark-up |
| Outsider damages - LB         | 0.12353        | 0.09412      | 0.15096      |
| Outsider damages - UB         | 0.42288        | 0.37230      | 0.47661      |
| Misallocation damages         | 0.26609        | 0.19664      | 0.33934      |
| Cartel damages                | 0.30961        | 0.27065      | 0.34453      |

| Mean damage per contract (whole white only)² $ |
| Outsider damages - LB         | 655.47         | 449.78       | 877.98       |
| Outsider damages - UB         | 1,902.96       | 1,425.27     | 2,378.19     |
| Misallocation damages         | 812.32         | 350.07       | 1,340.04     |
| Cartel damages                | 1,330.21       | 1,065.17     | 1,599.61     |

| Mean damage per contract³ $ |
| Outsider damages - LB         | 2,438.30       | 1,539.83     | 3,385.41     |
| Outsider damages - UB         | 7,074.81       | 4,911.70     | 9,181.49     |
| Misallocation damages         | 3,000.26       | 952.90       | 5,331.35     |
| Cartel damages                | 4,937.94       | 3,808.12     | 6,100.61     |

| Mean inefficiency due to Misallocation (efficient cartel) |
| per half-pint ($)         | 0.00394        | 0.00249      | 0.00560      |
| in percentage loss         | 0.03723        | 0.02494      | 0.06895      |
| per contract ($)           | 2,909.34       | 747.15       | 5,677.51     |
| per contract (whole white only) ($) | 788.55       | 294.23       | 1,430.84     |

Note: $ are 1982 dollars. UB corresponds to an inefficient cartel. LB corresponds to an efficient cartel. Only cartel damages in the case of an efficient mechanism are reported. For an inefficient mechanism, these damages are the same as the UB on outsider damages. ²per contract (whole white only): doesn't include damages for the other milk categories. ³per contract: computed by applying the overcharge estimated to the total quantity purchased (whole white and other categories). Proportions of auctions in the efficient cartel case: 33% with outsider damages, 9% with misallocation damages, 58% with cartel damages. Confidence intervals based on 2500 bootstrap iterations.
Figures

Figure 1: Map of the counties in the dataset, by market area
Figure 2: Estimates and 95% Confidence Intervals for the county dummies
Figure 3: Estimated bid functions for auctions with three bidders with an efficient and inefficient cartel mechanism

(a) Efficient cartel mechanism

(b) Inefficient cartel mechanism
A Algorithm for solving the asymmetric auction (Step 3)

The details of the numerical method used to solve the asymmetric auctions are presented here. Recall that the cartel and non-cartel bidders’ equilibrium inverse bid functions, denoted $\phi_1$ and $\phi_2$ respectively, are the solutions of:

\[
\begin{align*}
\frac{d\phi_i}{db} &= \frac{1-F_i(\phi_i)(b - \phi_i(b))}{f_i(\phi_i(b))} \\
\frac{d\phi_i}{db} &= \frac{1-F_2(\phi_2(b))(b - \phi_2(b))}{f_2(\phi_2(b))}
\end{align*}
\]  

(13)

where $F_i$ (resp. $f_i$) is the cumulative distribution (resp. density function) of costs of the cartel ($i = 1$) and the outsider firm ($i = 2$). Along with the boundary conditions $\phi_1(\underline{b}) = \phi_2(\underline{b}) = \underline{c}$ (for some $\underline{b}$, lower bound of the support of bids) and $\phi_1(\bar{c}) = \phi_2(\bar{c}) = \bar{c}$. The location of the left-boundary $\underline{b}$ is unknown. As shown in Fibich and Gavish (2011), the system of differential equations (13) can be recast as a system in $\phi_1$ and $b$ (instead of $\phi_2$) as functions of $\phi_2$. After this change of variable, the new BVP is given by:

\[
\begin{align*}
\frac{d\phi_1}{db} &= \frac{1-F_1(\phi_1)}{f_1(\phi_1)} f_2(\phi_2) \frac{b - \phi_1}{1-F_2(\phi_2)} - \phi_1 \\
\frac{db}{d\phi_2} &= \frac{f_2(\phi_2)}{1-F_2(\phi_2)} (b - \phi_1)
\end{align*}
\]  

(14)

with the boundary conditions: $\phi_1(\phi_2 = \bar{c}) = b(\phi_2 = \bar{c}) = \bar{c}$, and $\phi_1(\phi_2 = \underline{c}) = \underline{c}$. This BVP is defined on a known domain $\phi_2 \in [\underline{c}, \bar{c}]$. Fibich and Gavish (2011) propose fixed point iterations as one possible method for solving (14). Iterations are given by:

\[
\begin{align*}
\frac{d\phi_1}{db} &= \frac{1-F_1(\phi_1^{(k)})}{f_1(\phi_1^{(k)})} f_2(\phi_2) \frac{1}{1-F_2(\phi_2)} \phi_1^{(k+1)} = \frac{1-F_1(\phi_1^{(k)})}{f_1(\phi_1^{(k)})} f_2(\phi_2) \frac{b^{(k)}}{1-F_2(\phi_2)} \\
\frac{db}{d\phi_2} &= \frac{f_2(\phi_2)}{1-F_2(\phi_2)} \phi_2^{(k+1)} = -\frac{\phi_1^{(k+1)}}{f_2(\phi_2)} f_2(\phi_2) \phi_2
\end{align*}
\]  

(15)

with the boundary conditions: $\phi_1^{(k+1)}(\phi_2 = \bar{c}) = b^{(k+1)}(\phi_2 = \bar{c}) = \bar{c}$, and $\phi_1^{(k+1)}(\phi_2 = \underline{c}) = \underline{c}$. In the case of this particular empirical application, the initial guess used are: $\phi_1^{(0)}(\phi_2) = \phi_2$ and $b^{(0)}(\phi_2) = 0.9 + (\bar{c} - 0.9) / \bar{c} * \phi_2$. Although convergence of the fixed point iterations is not guaranteed, because a unique solution exists, if the algorithm converges to a function satisfying the BVP, this function is the solution to the BVP.