Secret Reserve Prices by Uninformed Sellers

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Abstract

If bidders are better informed than the seller about a common component of auction heterogeneity, the seller can allocate more efficiently by keeping her reserve price secret and revising it using submitted bids. We build a model of a first-price auction under unobserved auction heterogeneity—imperfectly observed by the seller—that captures this rationale and derive conditions for identification. An application to French timber auctions, where such revisions are widely used, shows that having perfect information about unobserved auction heterogeneity would increase surplus by 5.22%. Combining a secret reserve price with learning from submitted bids reduces this surplus gap by up to 84%.

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1 Introduction

Since the seminal work of Myerson (1981) and Riley and Samuelson (1981), the theoretical auction literature has focused on the optimal choice of a public reserve price, ignoring the empirical regularity that reserve prices are often kept secret. For instance, secret reserve prices are employed in auctions for fine art and wine (Ashenfelter (1989)), online auctions (Bajari and Hortaçsu (2003), Hossain (2008)), and auctions of natural resources such as oil or timber (Hendricks et al. (1989), Elyakime et al. (1997)).[1] The prevalence of secret reserve prices in real-world markets is at odds with insights from classical auction theory (e.g., in the first-price format with independent private values) suggesting that secret reserve prices are neither efficient nor optimal. This paper proposes a novel theoretical explanation for the use of secret reserve prices—the potential for bids to convey valuable information to the seller—and uses the French timber industry as an empirical application supporting this rationale.

If bidders are better informed than the seller about the underlying heterogeneity of the auctioned item, the seller can allocate more efficiently by keeping her reserve price secret and adjusting it after bids are submitted. Indeed, bids convey useful information about what the seller's reservation value should be. By contrast, when committing to a public reserve price, the seller loses the option value of learning from the bids: the uninformed reserve price may be higher than the highest bidder's valuation leading to no sale, even when an informed seller would have preferred to sell. The literature on strategic bid skewing provides strong evidence that bidders often possess more precise information about the ex-post realization of quantities in timber auctions (Athey and Levin (2001)) and in procurement of construction projects (Luo and Takahashi (2019), Bolotnyy and Vasserman (2019)) or information about future adaptation costs (Bajari et al. (2014)).

The main contribution of this paper is to provide a rationale for the use of secret rather than public reserve prices in environments where the auctioneer may value efficiency (i.e., government agencies) and faces some uncertainty about their reservation value.[2] Bids can then be used to refine the auctioneer's appraisal. In this sense, our explanation complements other rationales proposed in the literature. Methodologically, we emphasize the role of

[1]Other examples include the markets for used cars, real estate, and highway construction in the U.S. Among its best practices for procurement of public works, the OECD recommends not publishing reserve prices (OECD (2009)).

[2]Whereas our main application focuses on efficient auctions, the framework can be used to study optimal auctions where the seller is uncertain about their reservation value. Efficiency may also be of interest to private firms such as two-sided platforms. Platforms need to attract both buyers and sellers and, therefore, have an incentive to design auctions that match buyers with the most ***relevant*** seller (rather than simply the seller with the highest willingness to pay), as in Gomes (2014). We thank a referee for pointing this out.

unobserved auction heterogeneity that is imperfectly observed by the auctioneer and provide new identification results in the context of first-price auctions with secret reserve prices.[3]

Our approach is guided by two important features of our empirical setting, the French timber industry. First, in timber auctions, the Public Forest Service (Office National des Forêts, ONF) sets an ex-ante secret reserve price which can be revised down if no bid is above it: around 40% of auctioned tracts are sold at a bid under the ex-ante secret reserve price. This feature cannot be accounted for by previous rationales for secret reserve prices.[4] Discussions with ONF officers indicate that revisions are based on the distribution of submitted bids and occur, in particular, if bids suggest that the initial appraisal value overestimated the true unobserved heterogeneity of the tract.

Second, bidders possess more precise information than the ONF about tract heterogeneity. Tracts differ with respect to timber volumes, composition, location, harvesting conditions, etc. In advance of each sale, the ONF collects tract characteristics and shares them with prospective bidders via a sale booklet. Due to the large number of tracts surveyed and the ONF's limited resources, tract characteristics (such as volumes or quality) reported in the booklet are purely indicative and often imprecise (the ONF has no contractual obligation vis-a-vis reported volumes).[5] Bidders, therefore, have strong incentives to conduct their own "cruises" since the winner pays a lump-sum (or fixed-price) amount irrespective of actual timber volumes or quality.

We use data on ten sales of standing timber by the ONF that took place in the Grand Est region in the Fall of 2003.[6] We observe information on 2,262 tracts auctioned via first-price sealed bid auctions, including: bids and bidder identities, tract-level characteristics reported in the sale booklets, and, importantly, the ONF's ex-ante secret reserve price. An important feature of our data is that we can combine the latter variable with information about bids

[3]Our rationale echoes a result of Bulow and Klemperer (1996) (Footnote 22) who find that, if bidders have interdependent values, a seller benefits from waiting until after the auction ends to set a reserve price (via a take-it-or-leave-it offer). In our setting, bidders' valuations are not only correlated to each other, but also to the seller's reservation value through an unobserved heterogeneity component.

[4]Previous models where the seller perfectly knows her value either assume that the seller never revises the secret reserve price (after bids are submitted) or only revises the reserve price up. Larsen (2020) argues that a seller might accept an offer below their reserve price if they were uncertain about their valuation (or bidders' value distribution) when setting their reserve price, or if they held optimistic beliefs about auction prices.

[5]As an example, interviews conducted with bidders in Marty (2015) (in French) show that discrepancies between announced and actual timber volumes are quite common:

> See what the ONF announced for the domanial forest X. We have just finished exploiting it, it's the ONF clerk who cruised it [...] They announced 798 m^3 of oak. [...] We found exactly 500 m^3 and they announce 798 m^3!

[6]This dataset was collected by Costa and Préget (2004). See also Préget and Waelbroeck (2012).

received and whether the tract was sold to identify the instances where the ONF adjusts its initial reserve price down to accept the highest bid.

Reduced form analysis of the ONF's revision rule reveals that: (1) when the highest bid is above the ex-ante secret reserve price, the tract is always sold to the highest bidder; (2) when the highest bid is below the ex-ante secret reserve price, the probability of sale depends on the distribution of bids relative to the ONF's appraisal value, as well as, the number of bids received. To further investigate the effect of the ONF's revision rule on auction outcomes (i.e., revenue and surplus), we estimate a structural model that captures the main features of our empirical setting. In particular, the model can be used to compute the value to the ONF of acquiring better signals about the underlying heterogeneity of a tract, and to compare the current policy to alternative reserve price policies.

We develop a model where firms bid in a first-price auction with unobserved auction heterogeneity (e.g., timber quality and volume). The unobserved heterogeneity component enters both bidders and seller's values. While this component is perfectly observed by the bidders as is standard in models with unobserved heterogeneity, we allow (but do not impose) it to be imperfectly observed by the seller.[7] The seller sets an ex-ante secret reserve price based on her noisy signal of unobserved auction heterogeneity, and she can revise this reserve price flexibly after bids are submitted. We show that, if the seller's revision rule satisfies a homogeneity assumption (in bids and appraisal value), separability of the unobserved heterogeneity component and bidders' idiosyncratic private values carries to the equilibrium bid function as in the model of Krasnokutskaya (2011).

Under mutual independence of the unobserved heterogeneity component, bidders' idiosyncratic values, and the seller's noisy signal (or measurement error), the model is identified from information on bids, the seller's appraisal value, and allocation decisions (sold-unsold). The identification proceeds in three steps: first, by using the joint distribution of an arbitrary bid and the corresponding appraisal value, the distribution of unobserved heterogeneity, bidders' individual bid component, and the seller's signal can be identified. Intuitively, the common unobserved heterogeneity component is identified from the within-auction correlation between bids and the seller's appraisal value. Second, a bidder's probability of winning conditional on the unobserved heterogeneity component is obtained from the *unconditional* probability of winning (observed in the data), the distribution of winning bids, and the distribution of unobserved auction heterogeneity. Third, the distribution of bidders' idiosyncratic value component is derived from knowledge of the conditional probability of winning by

[7] Correlation in bids could be due to affiliation (i.e., factors that are unobserved to the bidders and the econometrician) or unobserved heterogeneity (i.e., auction-specific information commonly known among bidders but not the econometrician). In the context of timber auctions, the latter appears to be the main source of correlation.

inverting the first-order condition. The estimation procedure proposed in the paper follows the steps of the aforementioned identification argument.

We quantify the costs (in terms of revenue and surplus) of the ONF's imperfect information about tract-level unobserved heterogeneity and we assess the benefits derived from keeping the reserve price secret and learning from the bids. As a benchmark, we simulate the counterfactual first-best outcome: that is, assuming the ONF perfectly knows the unobserved heterogeneity component and sets a public reserve price equal to their true reservation value (efficient auction). This counterfactual gives an upper bound on total surplus and, importantly, allows us to compute the value to the ONF of acquiring better signals (value of information). Second, we compare the current policy to several alternatives: (a) no reserve price, (b) announcing a public reserve price, (c) setting an ex-post secret reserve price equal to a convex combination of average bid and ex-ante secret reserve price.

Counterfactual simulations show that acquiring perfect signals about unobserved heterogeneity would allow the seller to increase revenue by 5.77% and surplus by 5.22%. This finding is useful in its own right as it permits a cost–benefit analysis of more comprehensive cruises of tract characteristics by the ONF. The result speaks more generally to the importance of a seller's appraisal technology in auction markets. In the context of the ONF's timber auctions, the seller supplements their imperfect appraisal technology (i.e., cruises) with information revealed by the bids.

With respect to the effect of learning, announcing a public reserve price (policy (b)) increases revenue by 5.86% and reduces surplus by 2.34% relative to the current policy. Switching from a public reserve price to a secret reserve price with efficient learning (policy (c) with weights that maximize surplus) reduces the surplus gap (relative to the first-best) by 84%. Under efficient learning, surplus would increase by 5.84% and revenue would decrease by 4.92% compared to a public reserve price (policy (b)). By learning from the bids, the seller trades off greater allocative efficiency against lower revenue per auction. We evaluate the robustness of these predictions to the sale format, the presence of asymmetries, endogenous participation, and dynamics.

Our framework and empirical results will be useful to other auction markets. For instance, a common feature in government procurement is the auctioneer's right to reject bids above a certain threshold (e.g., the engineer's cost estimate of a highway repair project). In these instances, bidders are effectively facing a secret reserve price with a pre-announced lower bound. This feature can be accounted for in our model by assuming that the seller's noisy signal of unobserved heterogeneity is publicly disclosed to the bidder (along with other auction-level observables), but the seller's private (reservation) value component is not.

Finally, our theoretical framework improves on previous rationales for secret reserve prices by explicitly accounting for unobserved heterogeneity. Indeed, auction-specific information known to the bidders but not to the econometrician is common and can have important implications for auction outcomes. We show, in particular, that our rationale can be tested against and distinguished from alternative rationales relying on independent and affiliated private values models.

Related Literature. This paper contributes to three strands of the literature. The first strand concerns solutions to the secret reserve price "puzzle." Vincent (1994) develops an example where secret reserve prices can induce greater participation in second-price auctions with interdependent values. Li and Tan (2017) show that secret reserve prices can yield higher revenue in first-price auction with I.P.V. if bidders are sufficiently risk-averse.[8] Horstmann and LaCasse (1997) show that in a common value setting, sellers of high-value items can signal to potential bidders by using secret reserve prices when there are resale opportunities. Elyakime et al. (1994) and Eklöf and Lunander (2003) argue that while public reserve prices may be optimal, secret reserve prices yield higher sales which benefits the auctioneer when paid a percentage of sales. Ji and Li (2008) study multi-round auctions with a secret reserve price and find via numerical simulations that a secret reserve price can yield lower expected procurement costs than a public reserve price, when the mean of bidders' cost distribution exceeds the mean reserve price.

Within this literature, the closest papers to ours are, first, Olimov (2013) who argues that, in eBay auctions for used tractors, sellers use secret reserve prices to run unsuccessful auctions to learn bidders' willingness to pay and use this information in subsequent resale opportunities.[9] Second, Coey et al. (2020) argue that, in the context of online auctions, secret reserve prices allow the seller to observe more bids (first and second highest) and dynamically adjust her reserve price in future auctions. Our approach differs from these papers in two respects: the seller learns about the unobserved component of auction heterogeneity rather than bidder's private valuations (or distribution), and our rationale does not rely on dynamics or resale opportunities.[10]

Three recent contributions propose explanations based on non-standard or irrational agents: Rosenkranz and Schmitz (2007) study first-price and second-price auctions if agents have reference-based utility. Hossain (2008) studies a dynamic second-price auction where a fraction of bidders are uninformed and learn only whether their private valuation is above a posted price. Jehiel and Lamy (2015) use a competing auction environment with some buyers

[8] See also Brisset and Naegelen (2006) for a similar argument in English auctions.

[9] In the model of Olimov (2013), it is unclear why rational bidders would ever reveal their willingness to pay in such auctions if the seller does not intend to sell. See the discussion in Jehiel and Lamy (2015).

[10] A number of papers discuss secret reserve prices in the context of eBay auctions. See, for instance, Bajari and Hortaçsu (2003), Katkar and Reiley (2007), and Einav et al. (2015).

who do not have rational expectations about the distribution of reserve prices when kept secret. In our particular case, buyers are firms with at least some firm-specific component of value, contract sizes are typically small relative to firm size, and buyers are well-informed about tract heterogeneity, making explanations based on risk-aversion, irrational belief or signalling less appealing. Moreover, in contrast to Elyakime et al. (1994), the seller is the auctioneer in our setting.

Second, the paper contributes to the empirical literature on timber auctions. This literature encompasses studies of transaction costs and choice between unit-price and lump-sum format (Leffler and Rucker (1991)), post-auction bargaining between seller and bidders (Elyakime et al. (1997)), the effect of resale (Haile (2000)), mergers and preference programs (Brannman and Froeb (2000)), bid skewing in unit-price auctions (Athey and Levin (2001)), collusion (Baldwin et al. (1997)), endogenous entry (Athey et al. (2011), Li and Zheng (2012), Roberts and Sweeting (2016)), the presence of risk-aversion (Lu and Perrigne (2008), Campo et al. (2011)). The closest papers to our study are: Athey and Levin (2001) who highlight the importance of private information about auction heterogeneity in the U.S. Forest Service timber auctions; and Li and Perrigne (2003) and Perrigne (2003) who uses French timber auction data and analyze the revenue effects of secret versus public reserve prices. The latter paper allows for risk-aversion and shows that secret reserve prices can be revenue-enhancing.

Finally, we build on the literature on unobserved auction heterogeneity in first-price auctions. The various approaches developed for identification include: the control function approach (Campo et al. (2003), Roberts (2013), Balat et al. (2016)), the misclassification approach (Hu et al. (2013), Luo (2019)), and more recent approaches based on a quasi-control method (Compiani et al. (2019)) and mixture models (Kitamura and Laage (2018)). Haile and Kitamura (2019) provide an excellent survey. Our identification method is closest to the deconvolution approach of Li and Vuong (1998) and Krasnokutskaya (2011). Ignoring unobserved heterogeneity can have significant impact on structural estimates as found by Asker (2010), Krasnokutskaya (2011), Krasnokutskaya and Seim (2011). Closest to our application, Grundl and Zhu (2019) show that bidders' risk neutrality in timber auctions would be rejected if unobserved heterogeneity is not controlled for. We contribute to this literature by extending the identification results of Krasnokutskaya (2011) to settings with secret reserve prices.[11]

The paper is organized as follows. Section 2 provides a simple example that illustrates the main intuition of the paper. Section 3 gives background information about the ONF timber

[11] For ascending auctions, Freyberger and Larsen (2019) show that secret reserve prices can help identify the distribution of unobserved heterogeneity and values when the number of bidders is unknown.

sale program. Section 4 describes the data and present reduced form evidence on the ONF's secret reserve policy rule. Section 5 presents the model. Section 6 shows the identification and estimation results. Section 7 presents the counterfactual analysis. Section 8 concludes.

All proofs are included in Appendix A.

2 A Simple Example

Before presenting the model, it is instructive to consider an example. In this setting, the information structure is simple enough so that the seller learns perfectly from the bids. The example highlights the main intuition and key features entering the general model.

A seller (she) offers a single object for sale to n bidders via a first-price auction. The seller's reservation value, denoted Y, can take values in $\{\frac{1}{2}, \frac{3}{2}\}$. Bidder i's value is the sum of two components: the common component Y and a bidder-specific private value $X_i \sim U[-\frac{1}{2}, \frac{1}{2}]$, which is independent of Y and across bidders. Therefore, if Y equals $\frac{1}{2}$, bidder values are distributed $U[0,1]$; whereas if Y equals $\frac{3}{2}$, bidder values are distributed $U[1,2]$, as shown in Figure 1.

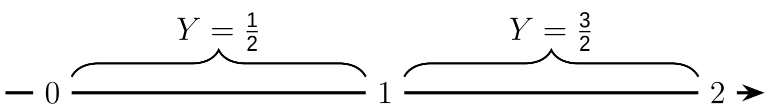

Figure 1

In addition to knowing his private value X_i, bidder i perfectly observes the common component Y. The seller, however, does not. Let the seller's prior belief about Y be uniform $\left(\frac{1}{2}, \frac{1}{2}\right)$. The seller acts as a social planner and aims to maximize total surplus.

In the first-best full information case, the seller perfectly observes Y and maximizes total surplus by setting an ex-post efficient public reserve price equal to Y. Ex-post surplus is equal to $\max\{Y, \max_i Y + X_i\}$.

Next, we compare surplus when the seller holds beliefs $\left(\frac{1}{2}, \frac{1}{2}\right)$ about Y and uses a public or secret reserve price. Under a public reserve price, expected surplus is maximized with a reserve price equal to $\frac{1}{2}$. If $Y = \frac{1}{2}$, the allocation is efficient. If $Y = \frac{3}{2}$, however, the item is always sold, which is inefficient if the highest bidder's value is less than $\frac{3}{2}$.[12]

[12]By setting a reserve price of $\frac{1}{2}$, the seller allocates efficiently when $Y = \frac{1}{2}$ but misallocates when $Y = \frac{3}{2}$. The benefits (relative to a reserve price of $\frac{3}{2}$) in terms of expected surplus is $\int_{\frac{1}{2}}^{1} \left(u - \frac{1}{2}\right) dH(u)$, where $H(u) = u^n$ is the distribution of the first-order statistic of bidder values ($Y = \frac{1}{2}$). The costs in terms of expected surplus is $\int_{1}^{\frac{3}{2}} \left(\frac{3}{2} - u\right) d\widetilde{H}(u)$, where $\widetilde{H}(u) = (u - 1)^n$ is the distribution of the first-order statistic of bidder values ($Y = \frac{3}{2}$). The benefits term dominates under a uniform prior.

With a secret reserve price, we argue that the seller can perfectly learn the value of Y from the bids and reach the first-best level of surplus. Consider the following equilibrium of the two-stage game in which bidders submit sealed bids, and given the bids and her prior belief about Y, the seller chooses an ex-post reserve price: In stage 2, if all bids are below 1, the seller sets the ex-post secret reserve price equal to $\frac{1}{2}$. Otherwise, the seller sets a reserve price equal to $\frac{3}{2}$. In the first stage, if $Y = \frac{1}{2}$, bidders bid as in a first-price auction with reserve price of $\frac{1}{2}$. if $Y = \frac{3}{2}$, bidders bid as in a first-price auction with reserve price of $\frac{3}{2}$.

In this example, not committing to a public reserve price allows the seller to delay her allocation decision until perfectly learning the true value Y from the bids. The allocation is ex-post efficient. In the general model of Section 5, we investigate the seller and bidders' behavior when the seller observes a noisy signal of Y and bids do not perfectly reveal the common component of value. Before doing so, the next section provides background information about the industry and the data that motivate features of the general model.

3 Industry Background

Our empirical work will focus on timber auctions conducted by the French national Public Forest Service (Office National des Forêts, ONF hereafter). This government agency is in charge of the management of France's approximately 11 million hectares of public forests and the sale of standing timber to mills and logging companies. Competitive bidding is the main mechanism chosen by the ONF for its timber sales (about 85% of total sales). We focus here on sales via (lump-sum) first-price sealed bid auctions, the most common auction format used by the ONF.[13]

Each administrative region in France has its own ONF local office. The data analyzed in this paper comes from the Grand Est (previously Lorraine) region (Eastern France). Local offices are in charge of the management of the public forestry on their own territory and are responsible for organizing auctions. Each regional office uses the profits from these sales to cover their operating costs. As the auctioneer, the ONF's objective is to secure timber supply to the local timber industry at a price that allows them to remain competitive. Therefore, we interpret the ONF's objective as the maximization of the local timber industry surplus subject to financial constraints (budget balance).

In advance of each sale, the ONF organizes "cruises" of the various tracts (around 200 per sale). A cruise consists in sending a team of prospectors to collect samples of the species present in a tract. Samples are then used to infer tract characteristics such as the

[13] Contrary to North American timber auctions, unit-price auctions where bidders submit a bid per species are less common.

composition, volumes per species, tree counts, tract surface, and condition of the trees. The ONF publicly announces the findings in a booklet available to potential bidders. Due to the large number of tracts surveyed and the ONF's limited resources, tract characteristics (in particular, volumes and quality) reported in the booklet are purely indicative and often imprecise. The ONF has no contractual obligation vis-a-vis reported volumes or quality.

Potential bidders are private firms, typically local sawmills. As the location of all auctioned tracts is given in the sale booklet, bidders have the opportunity to cruise the tracts and form their own estimates of tract characteristics. This is especially relevant given the lump-sum auction format used, where the winner pays his bid irrespective of realized timber volumes and composition. These cruises allow the bidder to gather two pieces of information: first, additional and more precise information about tract characteristics common to all firms (volumes, quality, etc.); second, information about their private value for the tract, which depends on firm-specific harvesting costs and the type of final product they will be able to sell using the harvested timber. Bidders' private values vary due to their diversity of operations: logging enterprises, sawmills, paper mills, board factories, etc.[14]

The ONF also computes an appraisal value for each tract which is not disclosed to the bidders, and based on this value, sets an (ex-ante) reserve price.[15] The reserve price is kept secret at the time of the auction. On the day of the sale, the ONF director collects sealed bids for each tract, opens the bids and ranks them. If the highest bid is above the ex-ante secret reserve price, the ONF sells the tract at the bid price. If the highest bid is below the ex-ante secret reserve price, the ONF may still decide to sell the tract to the highest bidder at their bid. The main criteria leading to a sale decision are: the number of bids received and their distribution, their difference relative to the ex-ante secret reserve price, and revenue constraints. About 40% of auctioned tracts are sold at a bid below the ONF's ex-ante reserve price. Whenever a tract is sold, the winner's identity and bid are publicly announced. If a tract goes unsold, the ex-ante secret reserve price is announced.

Discussions with ONF officers reveal that the Forest Service does not commit to any public reserve price because they do not perfectly know their reservation value. This value corresponds to the expected outcome in a future sale, which depends primarily on the tract characteristics, how market participants value each characteristics, and future timber market conditions. Recognizing that bidders have better information about tract characteristics, the ONF does not commit to a public reserve price in order to retain the option of adjusting its

[14] In 2018, of the 15 millions m^3 of timber sold, 4 were destined for construction, 3 for furniture manufacturing, 4 for the paper and cardboard industry, and 4 for energy.

[15] Discussions with the ONF reveal that this appraisal value does not account for bidder's private values (i.e., final product), but assess the value of timber as intermediary input entering each mills idiosyncratic production function. We interpret this appraisal value as reflecting the common component.

reservation value after observing the bids.

4 Data and Reduced-Form Analysis

This section describes the data used in our empirical application. We use data on ten sales of standing timber by the ONF that took place in the Grand Est region in the Fall of 2003. This dataset was collected in the context of a report commissioned by the ONF (Costa and Préget (2004)). The data contains information on 2,262 tracts auctioned via first-price sealed bid auctions, including: bids and bidder identifiers, tract level characteristics reported in the sale booklets (estimates of volume per species, surface, number of trees, etc.), the ONF's initial appraisal value for each tract, and the ONF's ex-ante secret reserve price. Over the sale season, $13,909$ hectares of land were auctioned for a gross revenue of $15,360,366$ €.

The data contains an array of tract characteristics, which helps control for auction heterogeneity. These tract characteristics are disclosed in the sale booklet to all prospective bidders. Descriptive statistics for the continuous and categorical variables are presented in Table 1 and Table 2 respectively. We refer the interested reader to Appendix B.1 for a more detailed description of the tract-level characteristics presented in Table 2. To capture tract level heterogeneity in volume per species, we construct a Herfindahl index of tract heterogeneity.

We analyze the main determinants of the bids and number of bidders in Table 3 via OLS regressions. Bids and participation are correlated with tract characteristics as expected: the volume of timber, homogeneity of species (Herfindahl index) are positively correlated with these outcomes variables. Tract quality (as controlled for by the categorical variables) has an expected sign: e.g., tracts with heavy grapeshot damages from WWI, or with difficult logging and extractions receive lower bids and attract fewer bidders.

A rare feature of the data is that we observe the seller's ex-ante secret reserve price as well as the allocation decision after bids are submitted. By combining the ex-ante secret reserve price with information about bids received and whether the tract was sold, we are able to identify the instances where the ONF adjusts its initial reserve price down to accept the highest bid.

Preliminary analysis indicates that revisions to the ex-ante reserve price are based on the bids received and the seller's appraisal value. Figure 2 shows a scatter-plot of auctioned tract sale status (i.e., sold or unsold) as a function of the highest bid (y-axis) and reserve price (x-axis) normalized by the appraisal value (or seller's estimate). The figure shows that tracts are always sold when the highest bid is above the ex-ante secret reserve price. Otherwise, the tract may or may not be sold when the highest bid is below the ex-ante reserve price. In

Variable	mean	std	min	max	r(rev)	p(rev)
Surface (ha)	11.34	12.21	0.2	299.0	-0.01	0.65
Trees (number)	239.86	204.18	18	2259	0.02	0.54
Poles (number)	195.21	529.51	0	11366	-0.05	0.09
Bidders (number)	2.46	2.39	0	13	0.23	0.0
Herfindahl index	0.63	0.21	0.21	1.0	0.0	0.97
Volumes (in m^3)						
Crown	121.59	134.04	0.0	1196.47	-0.02	0.56
Stump	0.23	4.74	0.0	153.83	0.01	0.64
Stem oak	59.06	101.3	0.0	859.98	0.01	0.68
Stem spruce	28.2	78.41	0.0	716.01	0.06	0.04
Stem beech	96.83	146.3	0.0	1365.8	-0.01	0.86
Stem pine	13.85	60.4	0.0	788.52	0.01	0.64
Stem fir	89.58	170.59	0.0	1240.98	0.02	0.48
Value (in euros)						
Reserve price	10886.71	10282.89	100	112000	0.02	0.5
Appraisal value	13154.41	12105.59	102	131662	0.03	0.29

Table 1: Descriptive statistics for the continuous variables. The last two columns show the correlation with the seller's decision to accept the highest bid, when all bids are below the ex ante reserve price. "r" and "p" stand for Pearson correlation coefficient and p-value.

particular, about half of tracts in which the highest bid is below the ex-ante reserve price, end up being sold.

In Table 4, we present the averages of the reserve price, highest bid, appraisal value, as well as the fraction of revisions among the tracts that were eventually sold, grouped by the number of bidders. The fraction of tracts sold after the reserve price was revised down increases with the number of bids received: if the highest bid is below the ex-ante secret reserve price, the tract is more likely to be sold when it attracted more bids.[16] This result echoes the last two columns of Table 1, showing the correlation between various tract characteristics and whether the reserve price is revised down. Except for the number of bidders, the decision to revise is not correlated with tract characteristics.

[16]This pattern is consistent with the seller putting more weight on bids when the number of bidders is high: in such instances, the signal that the seller receives through bids regarding auction-level unobserved heterogeneity is more precise.

Variable	Explanation	# tracts	% tracts	mean(rev)	std(rev)
Stand					
	High forest	1199	53.1	0.56	0.5
	Conversion of a stand	803	35.56	0.39	0.49
	Coppice forest	149	6.6	0.45	0.5
	Coppice with standards	107	4.74	0.5	0.5
Cut					
	Arranged Cutting	1222	54.12	0.52	0.5
	Regeneration Cutting	756	33.48	0.47	0.5
	Selection Cutting	164	7.26	0.41	0.5
	Other Cutting	71	3.14	0.55	0.51
	Accidental Products	45	1.99	0.42	0.51
Grapeshot					
	No damage	1649	73.03	0.52	0.5
	Light damage	373	16.52	0.37	0.48
	Average damage	150	6.64	0.5	0.5
	Heavy damage	57	2.52	0.15	0.38
Owner					
	Community-owned forest	1637	72.5	0.44	0.5
	State-owned forest (ONF)	621	27.5	0.63	0.48
Landing area					
	Unarranged	1914	84.77	0.51	0.5
	Arranged	277	12.27	0.43	0.5
	None	67	2.97	0.26	0.44
Quality					
	Average	930	41.19	0.48	0.5
	High	907	40.17	0.51	0.5
	Very low	240	10.63	0.41	0.49
	Very high	103	4.56	0.52	0.5
	Low	49	2.17	0.45	0.52
Conditions					
	Normal l&e	1380	61.12	0.5	0.5
	Easy l&e	492	21.79	0.44	0.5
	Difficult l&e	227	10.05	0.5	0.5
	Difficult extraction	69	3.06	0.57	0.5
	Very Difficult l&e	63	2.79	0.43	0.51

Table 2: Descriptive statistics for the categorial variables. The last two column shows the mean and standard deviation of the ONF's revision decision. "rev" stands for the binary decision to sell (1 vs 0), when all bids are below the ex ante reserve price, and "l&e" stands for logging and extraction. Grapeshot damage is damage from WWI.

Table 3: Determinants of bids and participation

	Bids (1) Estimate	S.E	# Bidders (2) Estimate	S.E
Number of bidders	0.268	(0.0116)		
Stand (ref: Conversion of stand)				
High forest	-0.0969	(0.0156)	0.00823	(0.0435)
Coppice forest	-0.0845	(0.0433)	-0.134	(0.0870)
Coppice with standards	-0.0408	(0.0223)	-0.0535	(0.0620)
Cut (ref: Arranged cutting)				
Other Cutting	-0.00990	(0.0338)	-0.106	(0.0714)
Selection Cutting	0.0970	(0.0381)	-0.0500	(0.0778)
Accidental Products	-0.308	(0.0642)	-0.455	(0.0939)
Regeneration Cutting	0.138	(0.0114)	-0.0225	(0.0301)
Grapeshot (ref: no damage)				
Light	-0.00639	(0.0160)	-0.252	(0.0352)
Average	-0.124	(0.0234)	-0.305	(0.0513)
Heavy	-0.139	(0.0384)	-0.371	(0.0795)
Owner (ref: community-owned)				
ONF-owned	0.0271	(0.0120)	0.148	(0.0317)
Landing area (ref: Arranged)				
None	-0.165	(0.0344)	-0.184	(0.0785)
Non-arranged	-0.0254	(0.0142)	-0.0618	(0.0401)
Quality (ref: High)				
Average	-0.0864	(0.0111)	-0.209	(0.0281)
Low	-0.0532	(0.0506)	-0.444	(0.0865)
Very Low	-0.137	(0.0192)	-0.208	(0.0447)
Very High	0.0446	(0.0191)	0.210	(0.0593)
Conditions (ref: Difficult L&E)				
Easy L&E	0.0562	(0.0215)	0.266	(0.0473)
Normal L&E	0.0861	(0.0193)	0.232	(0.0407)
Very difficult L&E	-0.205	(0.0413)	-0.00291	(0.0802)
Difficult E	0.0596	(0.0350)	0.144	(0.0787)
Continuous controls				
Herfindahl index	0.223	(0.0411)	0.358	(0.101)
Number of trees	-0.0501	(0.0151)	-0.0749	(0.0381)
Surface	-0.0294	(0.0116)	-0.115	(0.0264)
Number of poles	-0.100	(0.00345)	0.00127	(0.00888)
Timber volume (total)	1.123	(0.0185)	0.422	(0.0449)
Sale FEs	Yes		Yes	
Volumes per species	Yes		Yes	
Observations	5,483		2,218	
F-stat	677.3		33.0	
R^2	0.85		0.40	
Adjusted R^2	0.85		0.39	

Note: Unit of observation: bid in model (1) and auction in model (2). Standard errors are in parenthesis. Dependent variables and continuous controls are in log. L&E refers to logging and extraction.

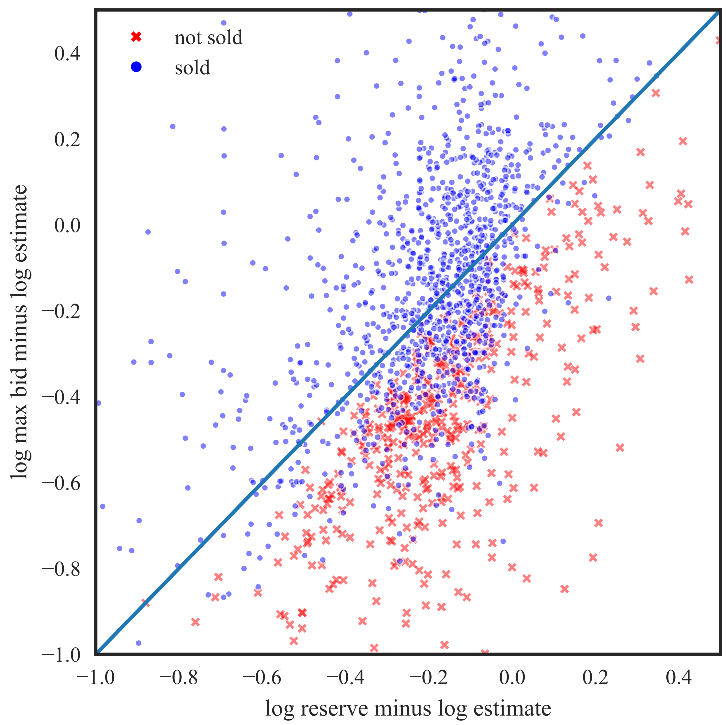

Figure 2: Scatter-plot of tract status (sold as green dots, unsold as red crosses) as a function of the highest bid (y-axis) and reserve price (x-axis) normalized by the appraisal value (seller's estimate). The 45 line corresponds to tracts where the maximum bid equals the (ex-ante) secret reserve price.

Number of bids	0	1	2	3	4-5	6+
Auctioned tracts	490	511	384	286	325	262
% tracts	0.22	0.23	0.17	0.13	0.14	0.12
% revise down	NA	0.32	0.51	0.58	0.65	0.7
avg log appraisal	8.77	8.9	9.07	9.24	9.46	9.72
avg log reserve	8.48	8.66	8.86	9.08	9.32	9.61
avg log max bid	NA	8.48	8.81	9.04	9.36	9.68
avg max bid / reserve	NA	0.89	1.0	0.99	1.09	1.12
avg reserve / estimate	0.78	0.81	0.84	0.88	0.89	0.91

Table 4: Descriptive statistics by number of bids submitted

Finally, we estimate a logit probability model for the revision decision (i.e., dummy for whether a tract is sold given that the highest bid is below the ex-ante reserve price) and report it in Table 5. We control for observed tract characteristics and show the coefficient on variables that a priori enter the ONF's decision to revise its ex-ante reserve price: i.e., the number of bids received, the highest bid, the average of the remaining bids, and the appraisal value.[17] In addition, we include interactions between theses variables and whether the tract is owned by the state (ONF-owned) or by a local commune. Specification (1) does not include controls for auction heterogeneity, specification (2) includes categorical controls, and specification (3) include both continuous and categorical controls for tract characteristics.

For tracts owned by the ONF, the revision decision depends positively on the number of bids received and the highest bid (as a fraction of the reserve price) and depends negatively on the mean bid (excluding the highest bid) and the appraisal value. This is consistent with the ONF learning: bids (and appraisal value) are used to form an ex-post reserve price that is compared to the highest bid. Consistent with the findings in the last two columns of Table 1 and Table 2, the continuous and categorical tract characteristics do not significantly predict the likelihood of revision given bids, appraisal value, and number of bids received.[18] We also verify that, once the ratio of bids to appraisal value is controlled for, the seller's revision decision does not depend on the actual *level* of bids and appraisal value.

5 The Model

This section presents the first-price auction model under secret reserve prices and unobserved auction heterogeneity. Bidders perfectly observe the common component of auction heterogeneity. The seller, however, only receives a noisy signal of the common component. We derive properties of the equilibrium bidding strategy in this context.

Random variables are denoted with upper case letters. Lower case letters denote realizations of random variables. Vectors are denoted in bold.

The seller (she) offers a single object for sale to n bidders in a first-price sealed bid auction. All players are risk-neutral and the number of bidders is common knowledge. The object is sold under unobserved auction heterogeneity. That is, bidder i's valuation is equal to the product of two components: one is common and known to all bidders; the other is individual and the private information of bidder $i \in \{1, ..., n\}$.[19] Both the common and the individual valuation components are random variables, and they are denoted by Y and X_i, respectively.[20]

[17] We also investigated alternative specifications using the second, and third highest bids.

[18] A likelihood ratio test of joint equality of the tract characteristics controls to zero cannot reject the null of equality to zero at the 1% level.

[19] Bidders' private values vary, in particular, due to their diversity of operations and depend on the profits from the final product made from the harvested timber.

[20] The assumption of multiplicative rather than additive separability is imposed as it is more consistent with patterns in the data. See the specification tests in Section 6.

Table 5: Logit Model for the Revision Decision

	Dep. variable: P(tract sold)		
	(1)	(2)	(3)
Number of bidders (log)	0.711	1.522	1.678
	(0.317)	(0.380)	(0.401)
ONF-owned	7.033	7.692	8.566
	(2.067)	(2.312)	(2.393)
ONF-owned Number of bidders (log)	-0.271	-0.709	-0.767
	(0.571)	(0.634)	(0.659)
Highest bid	10.46	11.36	11.68
	(1.154)	(1.265)	(1.329)
ONF-owned Highest bid	0.124	0.771	0.759
	(2.354)	(2.576)	(2.639)
Mean bid	2.126	2.725	2.989
	(0.951)	(1.097)	(1.165)
ONF-owned Mean bid	-3.436	-4.726	-5.141
	(2.035)	(2.210)	(2.254)
Appraisal value	0.955	0.110	0.277
	(0.497)	(0.567)	(0.611)
ONF-owned Appraisal value	-2.775	-2.356	-2.638
	(0.947)	(1.046)	(1.103)
Categorical controls	No	Yes	Yes
Continuous controls	No	No	Yes
Observations	1,257	1,240	1,240
Pseudo R^2	0.41	0.46	0.48
LR χ^2	543	611	637
Prob > χ^2	0.00	0.00	0.00
Baseline predicted probability	0.55	0.55	0.55

Note: Unit of observation: auction (with at least two bidders). Standard errors are in parenthesis. Bids and appraisal value are scaled by the reserve price. Mean bid is the average of all bids excluding the highest. Categorical controls include all the variables in Table 2 and dummies for each sale. Continuous controls include all timber volumes per species, Her ndahl index, surface, number of trees (in log) and lot order (within sale).

The seller's reservation value is her opportunity cost of selling the object. If a tract goes unsold, the seller can re-auction it the following year. Therefore, the seller's reservation value depends on the common component Y. Let the seller's reservation value be the product of the common component Y and a private component X_0.[21] In our setting, the seller is imperfectly informed about the realization of Y. We assume that the seller observes only a noisy signal of Y, denoted $\tilde{Y} = Y \quad S$, for some random variable S. The latter variable corresponds to a measurement error (in volumes and quality). In our empirical application, the variable $\tilde{Y}$ corresponds to the seller's private appraisal value.[22]

Information sets: The information set of bidder i is $\{x_i, y\}$. The seller's information set is $\{x_0, \tilde{y}\}$.

Primitives of the model: For simplicity, we assume that bidders' private values are symmetric and independent. The primitives are the marginal distribution of Y, X_i's, S, and X_0.

The following standard assumptions are maintained throughout:

1. All random variables Y, X_i's, S, and X_0 are assumed mutually independent, with marginal distributions denoted F_X, F_Y, F_S, and F_{X_0}.

2. The supports are given by $[\underline{x}, \overline{x}]$, $[\underline{y}, \overline{y}]$, $[\underline{s}, \overline{s}]$, and $[\underline{x}_0, \overline{x}_0]$. The lower bounds satisfy: $\underline{x} > 0$, $\underline{y} > 0$, $\underline{s} > 0$, and $\underline{x}_0 > 0$.

Reserve price: Before observing $\tilde{Y}$ and X_0, the seller commits to using an *ex-post* reserve price rule $R_1(\cdot)$ which takes the seller's idiosyncratic value X_0, the appraisal $\tilde{Y}$ (both kept secret at the time of the auction) and bids received $\mathbf{B} = (B_1, ..., B_n)$ as arguments.[23] The function $R_1(X_0, \widetilde{Y}, \mathbf{B})$ is common knowledge to all players. The object is allocated to the highest bidder if his bid exceeds R_1. In our empirical application, we do not impose an objective function for the seller nor do we assume that the seller is using Bayes' rule in setting the ex-post reserve price.[24]

We impose the following assumptions on the shape of the ex-post reserve price.

[21] The assumption that the ONF receives its reservation value when a tract is not sold (instead of no payoff) is consistent with the previous literature, e.g., Li and Zheng (2012), Roberts and Sweeting (2016). We interpret X_0 as the continuation value of keeping the tract unsold if $Y = 1$.

[22] We rule out any common-value dimension between bidders: that is, knowing bidder j's private information does not affect bidder i's valuation of the tract. However, because the seller observes only a noisy signal of Y, there is a common-value dimension between the bidders and the seller.

[23] We assume the range of (possibly off-equilibrium) bids to be $[0, \infty)$. If bidder i refrains from submitting a bid, the seller takes B_i to be equal to the minimal bid among the bids submitted. This is a technical assumption required to define the ex-post reserve price off-equilibrium path and, thus establish full participation on the equilibrium path.

[24] If the seller maximizes profits and cannot pre-commit, she would choose a secret reserve price equal to her expected reservation value given her information set (see, for instance, Elyakime et al. (1994) and Li and Tan (2017)). That is: $R_1(X_0, \widetilde{Y}, \mathbf{B}) = X_0 E[Y | \widetilde{Y}, \mathbf{B}]$.

Assumption 1. *The ex-post reserve price $R_1(x_0, \tilde{y}, \mathbf{b})$ is strictly positive, differentiable, weakly increasing in all arguments and strictly so in $\tilde{y}$, and the following conditions hold:*

1. *(Homogeneity of degree 1)* $\forall \lambda > 0,\ \ R_1(x_0, \lambda\tilde{y}, \lambda\mathbf{b}) = \lambda R_1(x_0, \tilde{y}, \mathbf{b})$
2. *(Sale guarantee)* $\forall x_0, \tilde{y}, \exists \bar{b} > 0 : R_1(x_0, \tilde{y}, \bar{b}, \ldots, \bar{b}) < \bar{b}$
3. *(No exclusion)* $R_1(\underline{x}_0, \underline{s}, \underline{x}, \ldots, \underline{x}) < \underline{x}$

Assumption 1.1 is required to prove the separability of bids in the unobserved heterogeneity component.[25] Assumption 1.2 means that, for a large enough bid, the tract will necessarily be sold. This assumption is required to prove existence of an ex-ante reserve price consistent with the allocation patterns in Figure 2 (see Lemma 1 below). Finally, Assumption 1.3 states that at the lowest realization of the seller's signal and private value, the tract will be necessarily sold. This assumption allows us to rule out binding reserve prices. Moreover, it is consistent with our discussions with ONF officers indicating that, in the vast majority of cases, all firms which cruise a tract end up submitting a bid (Athey and Levin (2001) document a similar pattern for the U.S. timber industry).

As described in the previous section, the ONF also chooses an *ex-ante* secret reserve price before bids are submitted. The allocation rule (Figure 2) is such that tracts are always sold when the highest bid exceeds this ex-ante secret reserve price. Note, however, that from the perspective of the bidders, only the *ex-post* reserve price is relevant to their bidding behavior. Appendix B.2 discusses the role of the ex-ante reserve price: in particular, the ONF uses this reserve price as a benchmark to simplify the auctioneer's allocation decision and is publicly announced at the end of the auction if all bids are rejected.

The following lemma: (i) characterizes the ex-ante reserve price, and (ii) derives an equivalent representation of the event where "bidder i out-bids the ex-post reserve price."

Lemma 1. *Under Assumption 1, there exist unique functions $R_0(x_0, \tilde{y})$ and $R_{-i1}(x_0, \tilde{y}, \mathbf{b}_{-i})$ defined as solutions to $R_1(x_0, \tilde{y}, R_0, \ldots, R_0) = R_0$ and $R_1(x_0, \tilde{y}, \mathbf{b})|_{b_i = R_{-i1}} = R_{-i1}$. These functions satisfy*

1. $b_i \geq R_0(x_0, \tilde{y}),\ b_i \geq \max_{j \neq i} b_j \quad \Rightarrow \quad b_i \geq R_1(x_0, \tilde{y}, \mathbf{b})$
2. $b_i \geq R_1(x_0, \tilde{y}, \mathbf{b}) \quad \Leftrightarrow \quad b_i \geq R_{-i1}(x_0, \tilde{y}, \mathbf{b}_{-i})$

for all i and $(x_0, \tilde{y}, \mathbf{b}_{-i})$. Moreover, R_{-i1} is continuous in all arguments and homogeneous of degree 1 in bids and appraisal value.

[25] As discussed in Section 4 (see Table 5), once the ratio of bids to appraisal value is controlled for, the seller's revision decision does not depend on the level of bids and appraisal value. This finding suggests that the allocation rule in the data is consistent with Assumption 1.1.

Lemma 1.1 shows that the ex-ante reserve price, defined as a fixed point of the ex-post reserve price rule when all bids are equal, satisfies the allocation rule shown in Figure 2. Lemma 1.2 states that, from the perspective of bidder i, playing against the ex-post reserve price rule is equivalent to playing against a random threshold R_{-i1}, that is only a function of the (random) rivals' bids and seller's valuation. In particular, the inequality $b_i > R_{-i1}(x_0, \widetilde{y}, \mathbf{b}_{-i})$ makes explicit that the probability of out-bidding the ex-post reserve price is an increasing function of b_i—a condition that guarantees that the second-order condition of optimal bidding holds, validating the first-order condition approach used in the estimation (see Appendix C for more details). In what follows, we use the two representations in Lemma 1.2 interchangeably.

Strategy and payoffs: Given the realization of the common component $y \in [\underline{y}, \overline{y}]$, a bidding strategy is a real-valued function defined on $[\underline{x}, \overline{x}]$:

$$\beta_y : [\underline{x}, \overline{x}] \rightarrow [0, \infty).$$

The profit realization of bidder i, $\pi(x_i, y; b_i)$, equals $(x_i y - b_i)$ if bidder i wins the tract with a bid b_i and zero if he loses. At the time of bidding, bidder i knows y and x_i but not his opponents's bids $\mathbf{B}_{-i} = \{B_j\}_{j \neq i}$ nor the seller's private information $(X_0, \widetilde{Y})$. The interim expected profit of bidder i is given by

$$\pi(x_i, y; b_i) = (x_i y - b_i) P(b_i \geq R_{-i1}(X_0, \widetilde{Y}, \mathbf{B}_{-i}) \cap b_i \geq B_j, j \neq i \mid Y = y) \tag{1}$$

To win the tract, bidder i must not only outbid his opponents but also the ex-post secret reserve price chosen by the seller. There are two sources of randomness to the ex-post secret reserve price: first, R_{-i1} depends on the seller's ex-ante valuation for the object $(X_0, \widetilde{Y})$ which differs from the known realization of the common component Y; second, R_{-i1} depends on bids submitted by bidder i's opponents. In the remainder of the paper, we refer to the probability in Equation (1) as the probability of winning conditional on $Y = y$ given a bid b_i, and to $P(b_i \geq R_{-i1}(X_0, \widetilde{Y}, \mathbf{B}_{-i}) \cap b_i \geq B_j, j \neq i)$ as the *unconditional* probability of winning given a bid b_i.

Equilibrium: We seek a symmetric Bayes-Nash equilibrium in continuous and strictly monotone strategies. The equilibrium is characterized by a function $\beta_y(.)$ such that $\pi(x_i, y; b_i)$ is maximized at $b_i = \beta_y(x_i)$, assuming that $b_j = \beta_y(x_j)$ for all $j \neq i$, all $i \in \{1, \ldots, n\}$ and realization of X_i. In the remainder of the paper, we assume that such an equilibrium exists.[26]

[26] The strict monotonicity and continuity properties are inherited by the bidding strategy from the probability of winning given bid b, by MCS Theorems (see Milgrom and Shannon (1994)) and by the Maximum Theorem, assuming uniqueness of the best response. In particular, Lemma 1.2 guarantees that as the player

Multiplicative separability: We extend a property of equilibrium bidding strategies under unobserved auction heterogeneity derived by Krasnokutskaya (2011) to the case with secret reserve prices.

Proposition 1. *Under Assumption 1, if $\alpha(.)$ is an equilibrium bidding strategy of the game indexed by $y = 1$, then an equilibrium bidding strategy in the game indexed by y, with $y \in [\underline{y}, \overline{y}]$, is such that $\beta_y(x_i) = y\alpha(x_i)$, for all i, with boundary condition $\alpha(\underline{x}) = \underline{x}$. Moreover, all types with private value above $\underline{x}$ have a strictly positive probability of winning.*

Proposition 1 states that separability of the unobserved heterogeneity component from private values carries to equilibrium bids. This property allows us to apply a deconvolution approach to separately identify private components from the common unobserved component as described in the next section.

6 Identification and Estimation

This section presents the identification of the model, the estimation approach, and the results.

6.1 Identification

In our setting, the econometrician has access to: bid data $(B_1, ..., B_n)$, the seller's appraisal value $\tilde{Y}$ and secret reserve price R_0, and auction outcomes. The data is based on N independent draws from the distribution of $(Y, \{X_i\}_{i=0...n}, S)$. We derive properties of the available data such that the model primitives are identified. For the rest of this section, the number of bidders is fixed to n.

Denote by $G_B(.)$ and $g_B(.)$ the cumulative distribution and density functions of the random variable B_i; and let b_{ij} denote the realization of B_i in auction j.

Proposition 1 shows that $b_{ij} = ya_{ij}$, where a_{ij} is the bid bidder i would submit if y were equal to one. We refer to a_{ij} as a normalized bid. We use A_i to denote the random variable with realizations equal to a_{ij} with distribution function denoted by $G_A(.)$ and probability density function $g_A()$. The variables y and a_{ij} are not observed by the econometrician.

The identification result is established as follows. First, it is shown that the probability density function of Y, A and S can be uniquely determined from the joint distribution of a bid and the seller's appraisal value. Second, the probability of winning, conditional on $Y = 1$ and given a bid a_i, is identified from the unconditional probability that bidder i wins the

increases their bid, the probability of exceeding the ex-post reserve price is non-decreasing. Consequently, the first order approach to characterizing the equilibrium is valid, see Appendix C for a more detailed discussion.

auction, the distributions of winning bids, and the distribution of the common component Y.[27] Third, monotonicity of the inverse bid function is used to identify the distribution of X_i from the distribution of A_i.

Proposition 2. *Under Assumption 1, the probability density functions f_Y, f_S, and f_X are identified from the distributions of bids, appraisal value, and allocation rule.*

The proof of this proposition consists of three steps. The first and third steps follow the arguments of Li and Vuong (1998) and Krasnokutskaya (2011). Our setting differs from the standard model of first-price auction, however, due to the seller's revision rule. Step 2 accounts for the seller's revision rule when expressing a bidder's probability of winning conditional on $Y = 1$. This probability is used in step 3 to invert the first-order condition and recover the distribution of bidder specific values.

Step 1: Identification of the probability density functions of Y, A_i, and S.
We apply the statistical result from Kotlarski (1966) to the log transformed random variables $B_i = A_i \quad Y$ and $\widetilde{Y} = S \quad Y$

$$\log(B_i) = \log(A_i) + \log(Y)$$

$$\log(\widetilde{Y}) = \log(S) + \log(Y)$$

Kotlarski's result shows that there is a mapping from the joint characteristic function of $(B_i, \widetilde{Y})$ (in logs) to the characteristic functions of the variable of interest A_i, Y, and S. Let $\Psi(.,.)$ and $\Psi_1(.,.)$ denote the joint characteristic function of $(\log(\widetilde{Y}), \log(B_i))$ and the partial derivative of this characteristic function with respect to the first component, respectively. Also, let $\Phi_{\log(Y)}(.)$, $\Phi_{\log(A_i)}(.)$, and $\Phi_{\log(S)}(.)$ denote the characteristic functions of $\log(Y)$, $\log(A_i)$, and $\log(S)$. Then,

$$\Phi_{\log(Y)}(t) = \exp\left(\int_0^t \frac{\Psi_1(0, u_2)}{\Psi(0, u_2)} du_2 \quad itE[\log(S)]\right)$$

$$\Phi_{\log(S)}(t) = \frac{\Psi(t, 0)}{\Phi_{\log(Y)}(t)}$$

$$\Phi_{\log(A)}(t) = \frac{\Psi(0, t)}{\Phi_{\log(Y)}(t)}$$

From the knowledge of the characteristic functions, we can derive the probability density functions of Y, S, and A. We first impose the normalization $E[\log(S)] = 0$.

[27] If the value one is not in the support of Y, then one can condition on any known arbitrary value y_0 in the support of Y instead. The support of Y is identified in the first step.

Step 2: Identification of the winning probability conditional $Y = 1$.

Let ω_i denote the event that bidder i wins the auction (unconditional of the realization of Y), that is,

$$\omega_i = \{B_i \geq R_1(X_0, \widetilde{Y}, B_i, B_{-i}) \cap B_i \geq B_{-i}\}.$$

Similarly, denote by $\widetilde{\omega}_i$ the event that bidder i wins the auction indexed by $Y = 1$

$$\widetilde{\omega}_i = \{A_i \geq R_1(X_0, S, A_i, A_{-i}) \cap A_i \geq A_{-i}\}.$$

In general, $\tilde{\omega}_i$ coincides with ω_i only conditional on $Y = 1$. Absent a reserve price (i.e., the first inequality $A_i \geq R_1(X_0, S, A_i, A_{-i})$), the probability of event $\widetilde{\omega}_i$ occurring is simply $1/n$, since bidders are symmetric. With a reserve price R_1, this probability will be less than $1/n$.

Let M denote the random variable A_i conditional on the event $\widetilde{\omega}_i$ and L denote the random variable B_i conditional on the event ω_i. Using Bayes' rule, the probability of winning conditional on $Y = 1$ and given a bid a, can be expressed as

$$P(\widetilde{\omega}_i | A_i = a) = \frac{g_M(a) P(\widetilde{\omega}_i)}{g_A(a)} \tag{2}$$

We show that the three probabilities on the right hand side of Equation (2) are identified from the data.[28] The density function of the normalized bids $g_A(a)$ has been identified in step 1. To identify the remaining two components, we rely on the following lemma.

Lemma 2. *Under Assumption 1, the event $\widetilde{\omega}_i$ is equal to the event ω_i. The distribution of M is identified from the deconvolution of the distributions of L and Y.*

Lemma 2 shows that the probability of event $\widetilde{\omega}_i$ is directly identified as the probability of winning an auction in the data, $P(\omega_i)$. Moreover, the distribution of normalized bids conditional on winning (M) is identified from the distribution of bids conditional on winning (L) and the distribution of Y. The characteristics function of L is identified from the data, while the characteristic function of Y is identified in the previous step. By independence of M and Y, we can recover the characteristic function of (log) M as

$$\Phi_{\log(M)}(t) = \frac{\Phi_{\log(L)}(t)}{\Phi_{\log(Y)}(t)}.$$

The characteristic function of M and its probability density function can be subsequently recovered from knowledge of the characteristic function of $\log(M)$.

[28] Absent a reserve price, Bayes' rule simplifies to the probability of outbidding rivals' bids: $P(\widetilde{\omega}_i | A_i = a) = \frac{n G_A(a)^{n-1} g_A(a) \frac{1}{n}}{g_A(a)} = G_A(a)^{n-1}$.

Noting that the event $\widetilde{\omega}_i$ can be equivalently expressed as

$$\widetilde{\omega}_i = \{A_i \geq R_{-i1}(X_0, S, A_{-i}) \cap A_i \geq A_{-i}\}$$

let $W = \max\{R_{-i1}(X_0, S, A_{-i}), A_{-i}\}$. In what follows, we denote the probability of winning conditional on $Y = 1$, given a bid a, as $F_W(a)$.

Step 3: Identification of the probability density functions of bidders' values X_i

We apply the result from Laffont and Vuong (1996) and Guerre et al. (2000) based on the first-order condition. Having recovered the probability of winning conditional on $Y = 1$ ($F_W(a)$), we can solve bidders' optimization problem and find the equilibrium inverse bidding strategy, that is,

$$\xi(a) \equiv \alpha^{-1}(a) = a + \frac{F_W(a)}{F'_W(a)}$$

The inverse bid function is combined with the distribution of normalized bids G_A (obtained in step 1) to back out the distribution of individual valuations X_i.

Step 4: Identification of the probability density function of the seller's private value X_0.

The fact that R_0 is observed in the data helps to identify the seller's private value X_0. Discussions with ONF officers revealed that R_0 is linear in the appraisal value, therefore, we impose the functional form $R_0(X_0, \widetilde{Y}) = X_0\widetilde{Y}$.[29]

From the equality $R_0 = X_0\widetilde{Y}$ and the independence of X_0 and $\widetilde{Y}$, the distribution of X_0 can be obtained by simple deconvolution of the (observed) distributions of R_0 and $\widetilde{Y}$.

6.2 Estimation

In our empirical application, tracts differ in observed dimensions (available to all bidders in the sale booklet). We control for this observed common component of heterogeneity in an initial step. The rest of the estimation approach follows the steps of the identification. The number of bidders is fixed to n.

1. Account for observed auction heterogeneity.

   The estimation procedure assumes that the data available is from auctions of ex-ante identical tracts. This assumption is not valid in our setting, because tracts differ in dimensions which are public information and observed by the bidders before submitting their bids (i.e., available in the sale booklet). This public information will enter not

[29] The choice $R_0 = X_0\widetilde{Y}$ is also consistent with the fact that the seller never revises up, that is $B_{(1)} \geq R_0 \Rightarrow B_{(1)} \geq R_1$, where $B_{(1)}$ is the highest bid. For certain specification of the function R_1, the choice of $R_0 = X_0\widetilde{Y}$ solves $R_0 = R_1(X_0, \widetilde{Y}, R_0)$. In Remark 1 of Appendix A, we characterize the family of functions R_0 that are consistent with the aforementioned revision pattern.

only a bidder's private value of winning the tract but also his belief about other bidders' values.

We follow the approach of Balat et al. (2016) to account for auction-specific heterogeneity. Their approach leverages the separability of common observable component from the bidder-specific and common unobserved heterogeneity components of bids. Let the seller's appraisal value in auction k be

$$\widetilde{y}_k = \Gamma(\mathbf{u}_k)\hat{y}_k$$

where $\Gamma(\mathbf{u}_k)$ is a function of the vector of observed auction characteristics $\mathbf{u}_k$ for auction k reported in the sale booklet and $\hat{y}_k$ is the seller's noisy estimate of unobserved heterogeneity. Similarly, let the value of bidder i in auction k be[30]

$$v_{ik} = \Gamma(\mathbf{u}_k)\hat{v}_{ik}$$

By multiplicative separability (Proposition 1), the corresponding bid of bidder i in auction j satisfies

$$b_{ik} = \Gamma(\mathbf{u}_k)\hat{b}_{ij}$$

Assumption 2 (Common observed heterogeneity)**.** *The observed common auction heterogeneity component enters identically into bidders and seller's values.*

Assume the following parametric specification: $\Gamma(\mathbf{u}_k) = \exp(\mathbf{u}_k'\delta)$.[31] We run a pooled first-stage regression, fixing the number of bidders, of the dependent variables $z_{ik} \in \{b_{ik}, \widetilde{y}_k\}$ on observed tract characteristics

$$\log z_{ik} = \mathbf{u}_k'\delta + \sigma_{ik} \tag{3}$$

where z_{ik} denotes the bid of bidder i in auction k and the seller's appraisal value and σ_{ik} is the error term. $\mathbf{u}_k$ include variables for tract surface, number of trees, number of poles, volumes per species, herfindhal index, sale dummy, order of the tract within the sale, and categorical variables (type of forest, type of cut, grapeshot damage, owner, type of landing area). All continuous variables are in logarithm. We recover the residuals $\log(\hat{b}_{ik}) = \log(b_{ik}) - \mathbf{u}_k'\widehat{\delta}$ (for the bidders) and $\log(\hat{y}_k) = \log(\tilde{y}_k) - \mathbf{u}_k'\widehat{\delta}$ (for

[30] Although the same function $\Gamma(\mathbf{u}_k)$ enters both bidder and seller's valuation and appraisal, the residuals $\hat{y}_k$ for the seller's appraisal value still contain the measurement error S, whereas the bidders' residual values $\hat{v}_{ik}$ do not.

[31] The log-linear relationship allows us to account for the skewness of bids and appraisal value expressed in nominal terms and obtain residuals symmetrically distributed around zero.

the seller). We refer to the residuals $(\hat{b}_{ik}, \hat{y}_k)$ as homogenized bids and appraisal values respectively.[32]

2. Separate the unobserved heterogeneity component from the bidder-specific component and seller's signal. We use the fact that homogenized bids and estimates obtained from the previous step are multiplicatively separable in the common unobserved component Y.

$$\log(\hat{b}_{ij}) = \log y_j + \log \beta(x_{ij}) \quad \text{and} \quad \log(\hat{y}_j) = \log y_j + \log s_j$$

   where $\beta(x_{ij})$ is the idiosyncratic component of bids attributable to variation in bidder's private valuations and s_j is the realization of the seller's signal in auction k. The joint characteristic function of an arbitrary bid and appraisal value (in logs) can be estimated as

$$\widehat{\Psi}(t_1, t_2) = \frac{1}{n \quad m} \sum_{i,j} \exp(it_1 \log(\hat{y}_j) + it_2 \log(\hat{b}_{ij}))$$

   where n is the number of bidders and m is the number of auctions (with n bidders). Next, the characteristic functions of the marginal distributions $(\log(Y), \log(S), \log(A))$ can be recovered as

$$\widehat{\Phi}_{\log(Y)}(t) = \exp\left(\int_0^t \frac{\widehat{\Psi}_1(0, u_2)}{\widehat{\Psi}(0, u_2)} du_2 \quad itE[\log(S)] \right)$$

$$\widehat{\Phi}_{\log(S)}(t) = \frac{\widehat{\Psi}(t, 0)}{\widehat{\Phi}_{\log(Y)}(t)} \quad \text{and} \quad \widehat{\Phi}_{\log(A)}(t) = \frac{\widehat{\Psi}(0, t)}{\widehat{\Phi}_{\log(Y)}(t)}$$

   where $\widehat{\Psi}_1$ is the derivative of the joint characteristic function with respect to its first argument. The normalization $E[\log(S)] = 0$ is first imposed.

   Densities are recovered using the inverse Fourier transform

$$\widehat{f}_{\log(Z)}(z) = \frac{1}{2\pi} \int_{T}^{T} d(t) \exp(\quad itz) \widehat{\Phi}_{\log(Z)}(t) dt \tag{4}$$

   where $Z \in \{A, Y, S\}$, T is a smoothing parameter, and $d(t)$ is a damping function (the choice of T and $d(t)$ are discussed at the end of this section).

[32] Assumption 1 and Assumption 2 imply that bids and reserve price scale linearly in $\Gamma(\mathbf{u}_k)$ and y. This ensures that the homogeneization approach used here would remain valid in auction markets where the reserve price is binding and only some bids are observed. Indeed, in such instances, the screening threshold would not depend on $\mathbf{u}_k$. We thank a referee for pointing this out.

Finally, the densities of de-logged variables Z are recovered as

$$\widehat{f}_Y(y) = \frac{\widehat{f}_{\log(Y)}(\log(y))}{y}, \quad \widehat{f}_S(s) = \frac{\widehat{f}_{\log(S)}(\log(s))}{s}, \quad \widehat{g}_A(a) = \frac{\widehat{g}_{\log(A)}(\log(a))}{a}$$

3. Estimate the probability of winning conditional on $Y = 1$. From Equation (2) and Lemma 2, the probability of winning conditional on $Y = 1$ can be estimated from the unconditional probability of winning $P(\omega_i)$, the distribution of residualized bids conditional on winning $\widehat{B}_i|\omega_i$ and the distribution of Y. From Equation (2), we define an estimator of the conditional probability of winning with a bid a as

$$\frac{\widehat{g}_A(a|\widetilde{\omega}_i)\widehat{P}(\widetilde{\omega}_i)}{\widehat{g}_A(a)} \tag{5}$$

where $\widehat{g}_A(a)$ is estimated in the previous step and $\widehat{P}(\widetilde{\omega}_i)$ can be estimated directly from the data as $\widehat{P}(\omega_i)$. The distribution of $\widehat{g}_{A_i}(a|\widetilde{\omega}_i)$ is obtained from the deconvolution of the estimated distribution of $\widehat{B}_i|\omega_i$ and the estimated distribution of Y. Denote $L = \widehat{B}_i|\omega_i$, with c.d.f $F_L(b) \quad \widehat{P}(\widehat{B}_i \quad b|\omega_i)$, and $\widehat{Q}_{\log(L)}(p)$ the quantile function of the random variable $\log(L)$. The characteristic function of $\log(L)$ is estimated as

$$\widehat{\Phi}_{\log(L)}(t) = \int_0^1 \exp(it\widehat{Q}_{\log(L)}(p))dp$$

Next, from $\log(L) = \log(M) + \log(Y)$, the characteristic function of $\log(M)$ is estimated from knowledge of the characteristic functions of $\log(L)$ and $\log(Y)$:

$$\widehat{\Phi}_{\log(M)}(t) = \frac{\widehat{\Phi}_{\log(L)}(t)}{\widehat{\Phi}_{\log(Y)}(t)}$$

The density and cumulative distributions of M are recovered from $\widehat{\Phi}_{\log(M)}(t)$ (by the inversion formula, as in step 2). Finally, denote by $\widehat{F}_W(a)$ the estimated probability of winning conditional on $Y = 1$.[33]

4. Recover the distribution of idiosyncratic values X_i and equilibrium bid function. Conditional on $Y = 1$, an estimate of the inverse bid function is obtained from the first-order condition

$$x = \widehat{\xi}(a) = a + \frac{\widehat{F}_W(a)}{\widehat{f}_W(a)}$$

[33] The probability $\widehat{P}(\widetilde{\omega}_i|A_i = a)$ is defined on the support of A_i. The boundaries of this support are estimated as described at the end of this section.

Denote by $\widehat{\alpha} = \widehat{\xi}^{-1}$ the corresponding estimate of the equilibrium bid function.

Finally, the distribution of private values is estimated by applying the distribution of bids (conditional on $Y = 1$), obtained in step 2, to the equilibrium bid function.

$$\widehat{F}_X(x) = \widehat{G}_A(\widehat{\alpha}(x))$$

Confidence intervals can be computed for all inferred values by bootstrap sampling at the auction level.

Practical considerations. A number of practical issues need to be addressed to perform the previous estimation. To implement the inverse Fourier transform (Equation (4)), we use a damping function to control fluctuations in the tail of the characteristic functions. Following Diggle and Hall (1993), we use the function

$$d(t) = \max\left(0, 1 - \frac{|t|}{T}\right)$$

For each random variable in $\{A, Y, S\}$, the smoothing parameter T is chosen to match empirical moments of these variables. We use the first and second moments:

$$\widehat{\mu}_{LS} = 0 \ \ , \widehat{\mu}_{LY} = \overline{\log(\hat{y}_k)} \ \ , \widehat{\mu}_{LA} = \overline{\log(\widehat{b}_{ik})} - \widehat{\mu}_{LY}$$
$$\widehat{\sigma}^2_{LA} = \widehat{\sigma}^2_{LB} - \widehat{\sigma}^2_{LY} \ \ , \widehat{\sigma}^2_{LS} = \widehat{\sigma}^2_{L\widetilde{Y}} - \widehat{\sigma}^2_{LY} \ \ , \widehat{\sigma}^2_{LY} = \frac{\widehat{\sigma}^2_{LB_i} + \widehat{\sigma}^2_{LB_j} - \widehat{\sigma}^2_{LB_i - LB_j}}{2}$$

For each random variable $Z \in \{A, Y, S\}$, T is chosen to minimize

$$\frac{(\widehat{\mu}_{LZ} - \widetilde{\mu}_{LZ})^2 + (\widehat{\sigma}^2_{LZ} - \widetilde{\sigma}^2_{LZ})^2}{\widehat{\sigma}^2_{LZ}} + P_{\chi\{non-monotonic\}}$$

where $P_{\chi\{non-monotonic\}}$ is a penalty function that deter the search from candidate smoothing parameters yielding negative values for the density of LZ. In practice, we obtain values of T for $\{Y, S, A\}$ equal to 14.0, 12.5, and 15.5 respectively.

Density estimates from the procedure in Step 2 suffer from being imprecise in the tails in finite samples. This leads to small positive densities being inferred over a very wide support. This problem is dealt with as follows: the support boundaries of the random variables obtained in step 2 ($[\underline{a}, \overline{a}]$, $[\underline{y}, \overline{y}]$, $[\underline{s}, \overline{s}]$) are estimated by combining the support of variables observed in the data and restrictions imposed by the model. In particular, we use the following restrictions

$$
\begin{cases}
\log(\widehat{\underline{b}}) = \log(\underline{a}) + \log(\underline{y}) \quad \text{and} \quad \log(\widehat{\overline{b}}) = \log(\overline{a}) + \log(\overline{y}) \\
\log(\widehat{\underline{y}}) = \log(\underline{s}) + \log(\underline{y}) \quad \text{and} \quad \log(\widehat{\overline{y}}) = \log(\overline{s}) + \log(\overline{y}) \\
\max\limits_{i,j,k} f \log(\widehat{b}_{ik}) \quad \log(\widehat{b}_{jk}) g = \log(\overline{a}) \quad \log(\underline{a}) \\
\int_{\log(\underline{s})}^{\log(\overline{s})} y \quad f_{\log(S)}(y)dy = 0 \quad (E[\log(S)] = 0)
\end{cases}
\tag{6}
$$

where $(\widehat{\underline{b}}, \widehat{\overline{b}})$ and $(\widehat{\underline{y}}, \widehat{\overline{y}})$ are estimates of the support boundaries of homogenized bids and seller's appraisal values. This system of equations uniquely determines the unknown support boundaries $[\underline{a}, \overline{a}]$, $[\underline{y}, \overline{y}]$, and $[\underline{s}, \overline{s}]$ (see Appendix A.2 in Krasnokutskaya (2011)). Estimates of the support boundaries of the normalized bids and appraisal values and the normalization $E[\log(S)] = 0$ allow us to recover these unknowns.

6.3 Estimation results

The results presented below correspond to auctions that attracted three bidders (283 tracts in total). The results for different values of the number of bidders are qualitatively similar.

The cumulative distributions of unobserved auction heterogeneity, seller's signal and private valuation, and individual bid component are represented in Figure 3. The distribution of unobserved heterogeneity has a mean of 1.09 and a standard deviation of 0.32. After incorporating observed auction heterogeneity ($\Gamma(\mathbf{u}_k)$), the mean and standard deviation of the common component are equal to 17,333e and 26,224 e, respectively. The recovered distribution for the seller's signal (or measurement error) has a standard deviation of 0.24. Taken together with the standard deviation in unobserved heterogeneity, the seller's appraisal value is a relatively noisy signal of the true realization of unobserved heterogeneity.

The distribution of the seller's private value indicates that the ex-ante secret reserve price is on average equal to 0.84 of the ex-ante appraisal value. One reason is the ONF discounting the value of keeping the item until the next sale season.

The variance of bidders' values $(X_i \quad Y)$ can be decomposed into the variance due to the unobserved auction heterogeneity and the variance due to idiosyncratic private value.

$$
\text{Var}(XY) \quad E[X^2]\text{Var}(Y) + E[Y^2]\text{Var}(X)
$$

Unobserved auction heterogeneity explains 36% of the variance in bidder values. Failing to control for this unobserved common component would have resulted in over-estimates of

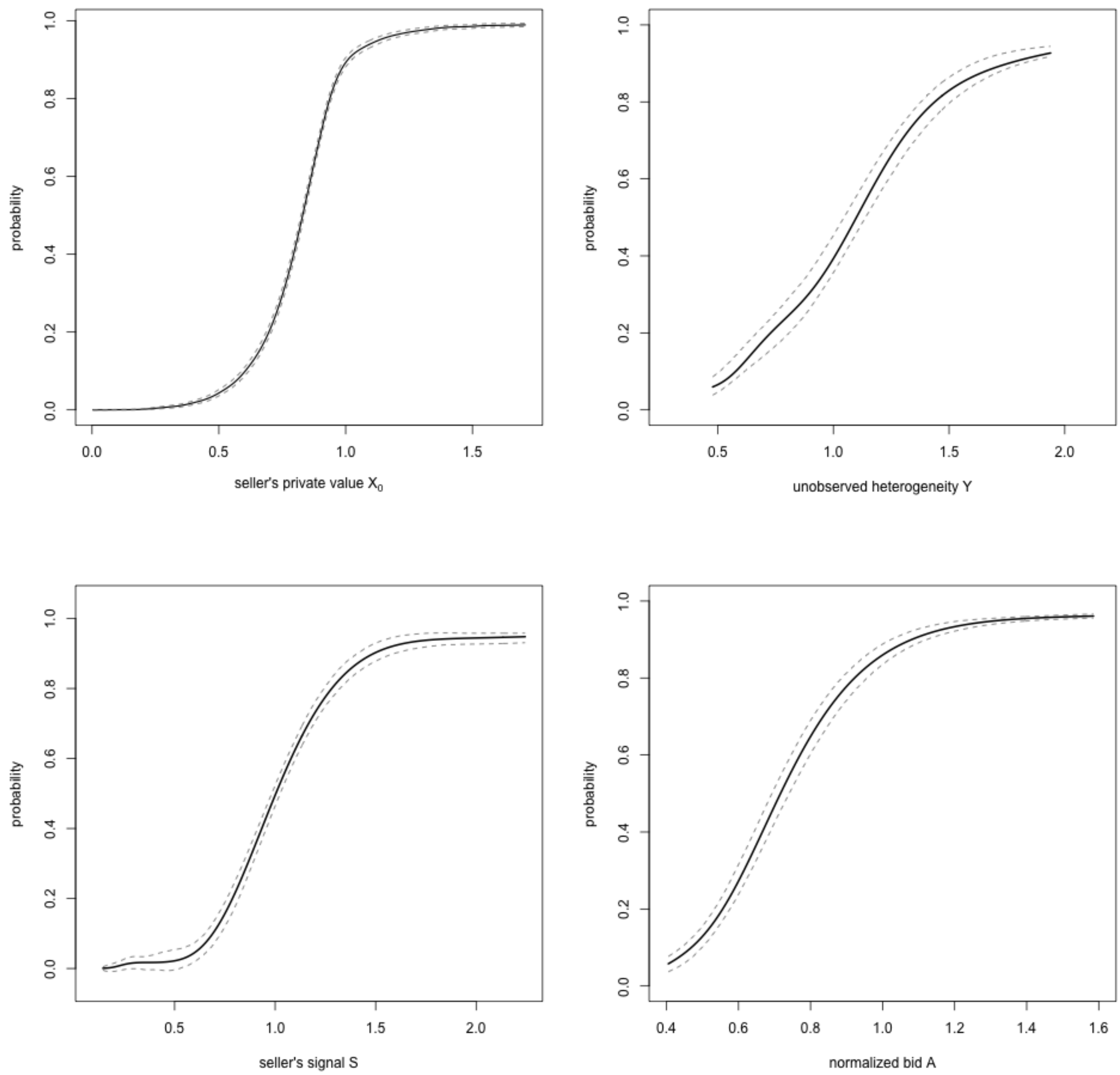

Figure 3: Cumulative distribution functions of the seller's private value, the unobserved auction heterogeneity component, the seller's ex-ante signal, and the individual bid component. The dotted lines show pointwise 95% confidence intervals estimated through a bootstrap procedure.

the variance of idiosyncratic private values.

Figure 4 shows the estimated equilibrium bid function (conditional on $Y = 1$) and the distribution of individual private valuation. The estimated bid function is used to compute mark-downs: bidders shade their bids by 15.6% on average below their value.

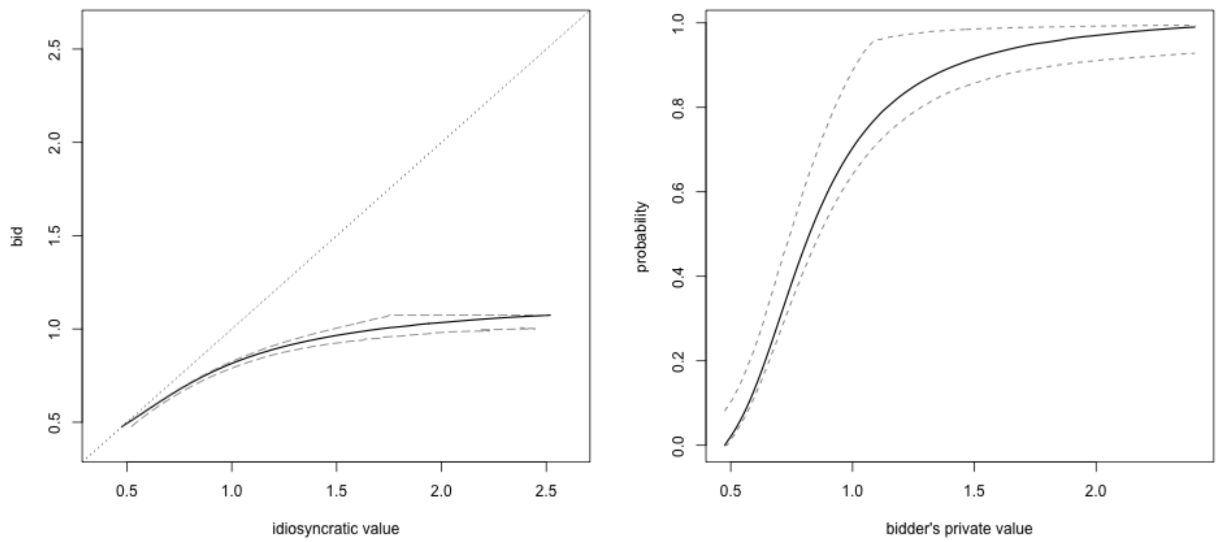

Figure 4: Estimated bid function conditional on $Y = 1$ (left) and estimated cumulative distribution of bidders' private values (right). The dotted lines show pointwise 95% confidence intervals estimated through a bootstrap procedure.

6.4 Specification tests

The model with unobserved heterogeneity implies a number of testable implications. We perform these specification tests here.

Test against APV models. Both the model with unobserved heterogeneity and the APV model imply correlation in bids. However, the unobserved heterogeneity model implies that bids are conditionally independent, whereas most APV models imply that bids are affiliated (see Krasnokutskaya (2011)).

To distinguish the model with unobserved heterogeneity from an APV model, we test for the independence of bid ratios formed from a quadruple of bids submitted in the same auction.[34] Under unobserved heterogeneity, the pairwise ratios should be independent. This property does not hold for a large class of APV models.[35] Figure 5 (top) shows density estimates as well the correlation between pairwise bid ratios (within-auction). The Spearman (rank) correlation coefficient equals 0.049 and the p-value for the test of zero rank correlation equals 0.27, so that the null hypothesis (of no monotone dependence between the two bid ratios) cannot be rejected.[36] We interpret this finding as strong evidence in favor of the

[34] We use four-bidder auctions to conduct this test.

[35] Because the set of affiliated distribution includes the set of conditionally independent distributions, this test has no power against certain APV models, as noted by Kranokutskaya (2011).

[36] The Pearson correlation coefficient equals 0.03 and the p-value for the test of zero correlation is 0.49.

model with unobserved heterogeneity against APV models. We perform the same exercise but using a bid ratio $\frac{B_1}{B_3}$ and a bid to reserve price ratio $\frac{B_3}{R_0}$ (within-auction) as shown in figure Figure 5 (middle). If the unobserved component enters linearly into the reserve price, the two variables should again be independent. The Spearman (rank) correlation coefficient equals   0.015 and the p-value for the test of zero rank correlation equals 0.729, so that the null hypothesis cannot be rejected.

Test against IPV models. Under an IPV model, bids should be independent across bidders, conditional on observed auction characteristics. We construct residual bids by regressing bids on a linear index of observed auction characteristics (as in step 1 of the estimation procedure). Figure 5 (bottom) shows density estimates as well as the correlation between residualized bids (within-auction). The Spearman (rank) correlation coefficient equals 0.726 and the p-value for the test of zero rank correlation equals 0.0, so that the null hypothesis of no correlation can be rejected.[37] This provide evidence against an IPV model, i.e., bids are correlated through an unobserved common component.

Other testable implications. The primitive density functions can be estimated using alternative pairs of variables.

First, Kotlarski's result can be applied to a pair of bids (b_{ik}, b_{jk}) submitted in the same auction (as in the procedure of Krasnokutskaya (2011)). Since $\log(B_{ik}) = \log(Y) + \log(A_{ik})$, the distribution of A and Y can be recovered. If the signal S is independent of X_i and Y, then the distributions recovered should be identical to the ones obtained using a pair of bid and appraisal value $(b_{ik}, \widetilde{y}_k)$.

Figure 6 presents density estimates under the alternative estimation approach along with the estimates using the baseline approach. In both cases, we impose the restriction that $E[\log(A)] = 0$. We test for the equality of each pair of density functions.[38] The p-values for the test of equality of the densities across the two estimation approaches are 0.10, 0.91, and 0.87 respectively. We cannot reject the null of equality of the three densities at the 5% confidence level.

Second, Kotlarski's result can be applied to a pair of bid ratios $(\frac{b_{1k}}{b_{3k}}, \frac{b_{2k}}{b_{3k}})$ (for auctions with at least three bidders). Since $\log(\frac{B_{1k}}{B_{3k}}) = \log(A_{1k}) \quad \log(A_{3k})$ and $\log(\frac{B_{2k}}{B_{3k}}) = \log(A_{2k}) \quad \log(A_{3k})$, the distribution of A can be recovered. If Y is independent of X_i, then the distribution of normalized bids recovered should be identical to the one obtained using a pair of bid and appraisal value $(b_{ik}, \widetilde{y}_k)$.

Figure 7 presents density estimates under the alternative estimation approach along with the estimates using the baseline approach. In both cases, we impose the restriction that

[37] The Pearson correlation coefficient equals 0.724 and the p-value for the test of zero correlation is 0.0.

[38] We implement the test proposed in Appendix A.3 in Krasnokutskaya (2011).

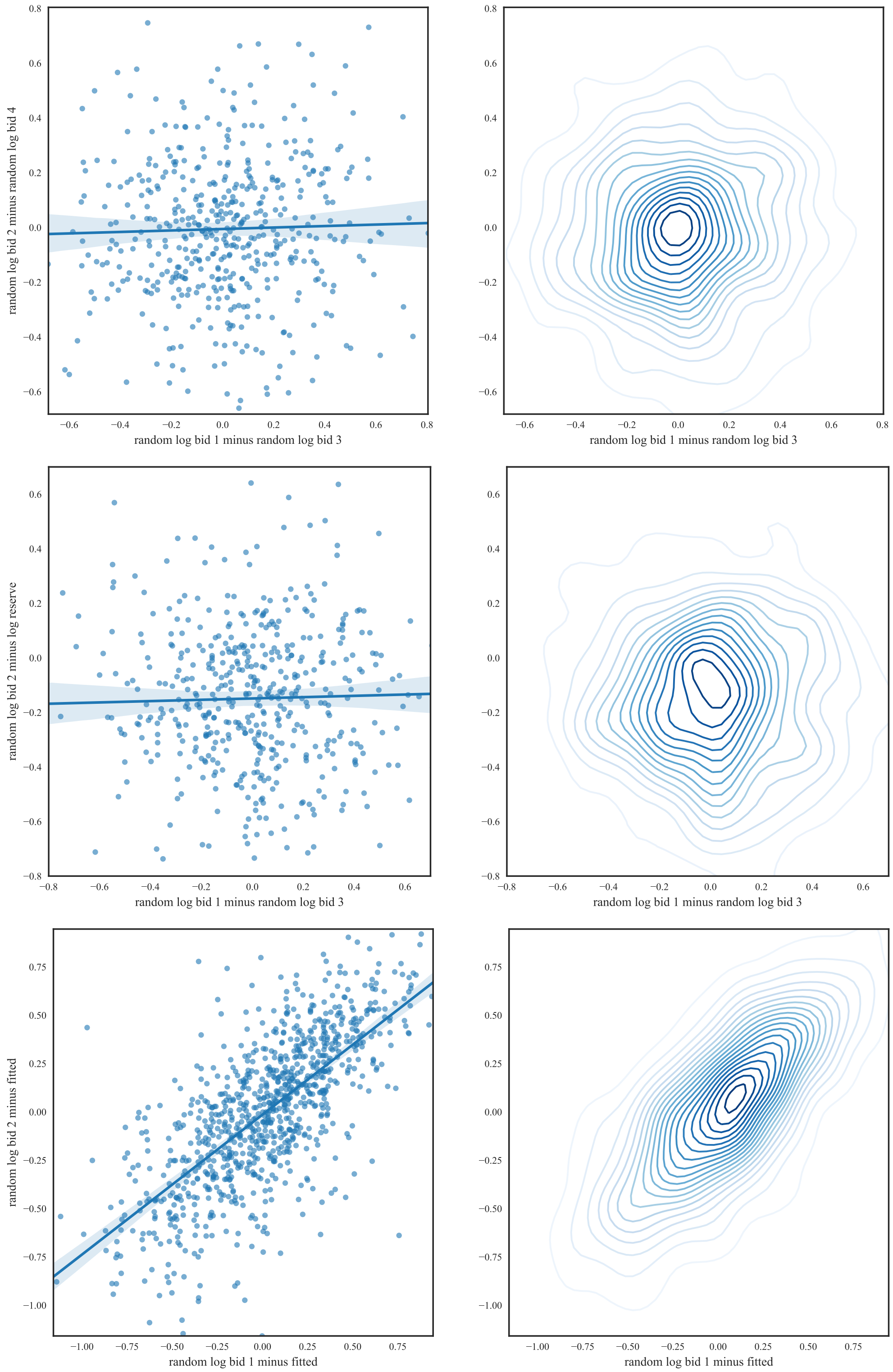

Figure 5: Scatter-plots and density estimates of (within-auction) two bid ratios (top), one bid ratio and bid to reserve price ratio (middle), two residual bids (bottom), all in logs.

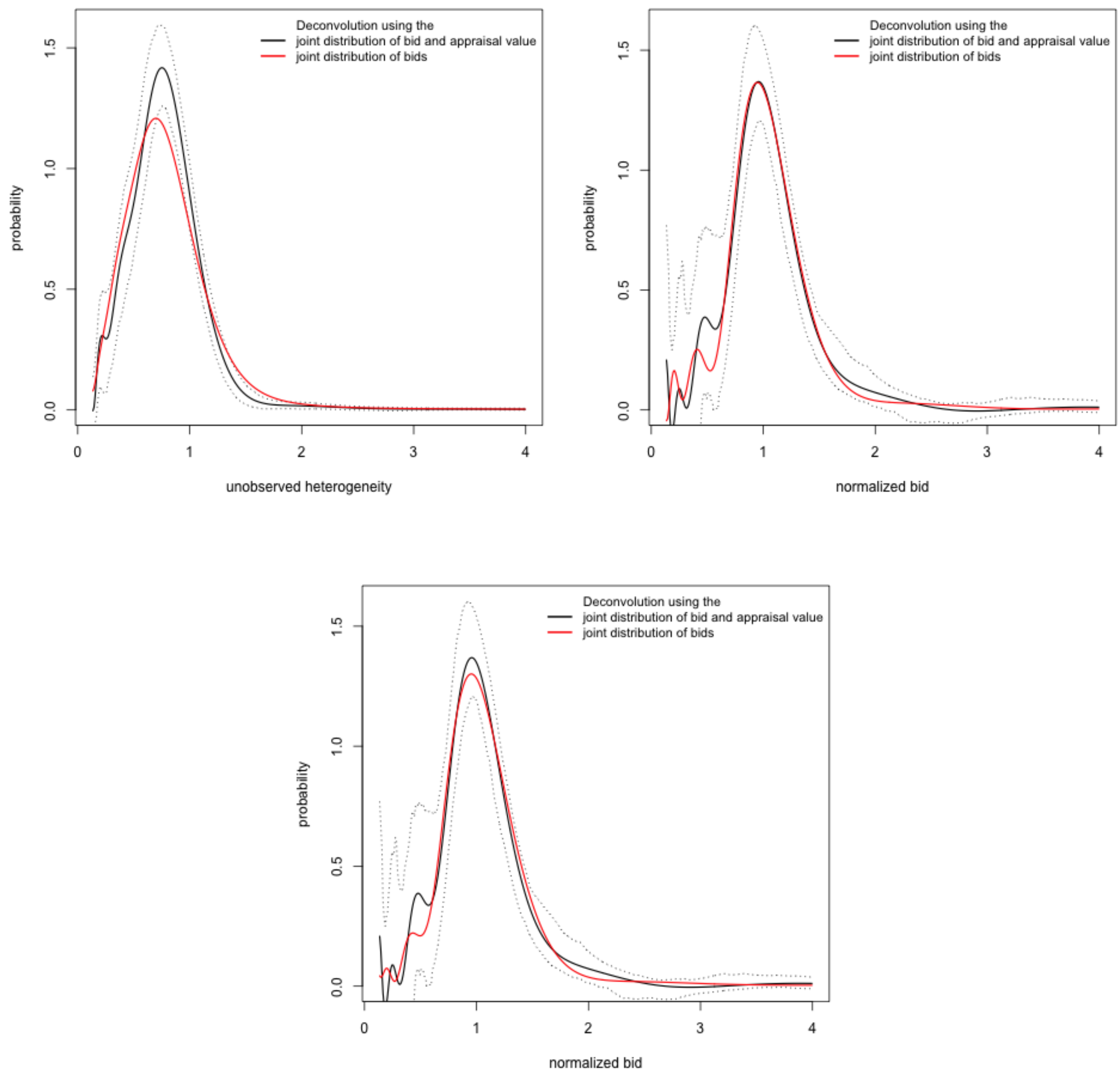

Figure 6: Estimated densities of the unobserved auction heterogeneity component and the individual bid component using (1) the joint distribution of bid and appraisal value, and (2) the joint distribution of two bids. The dotted lines show pointwise 95% confidence intervals for (1), estimated through a subsampling procedure.

$E[\log(A)] = 0$. The p-values for the test of equality of the densities across the two estimation approaches are 0.85, 0.90, and 0.88 respectively. We cannot reject the null of equality of the three densities at the 5% confidence level.

Taken together, the specification tests show that the data supports (1) the presence of unobserved heterogeneity entering as a separable component into bidders' values and the

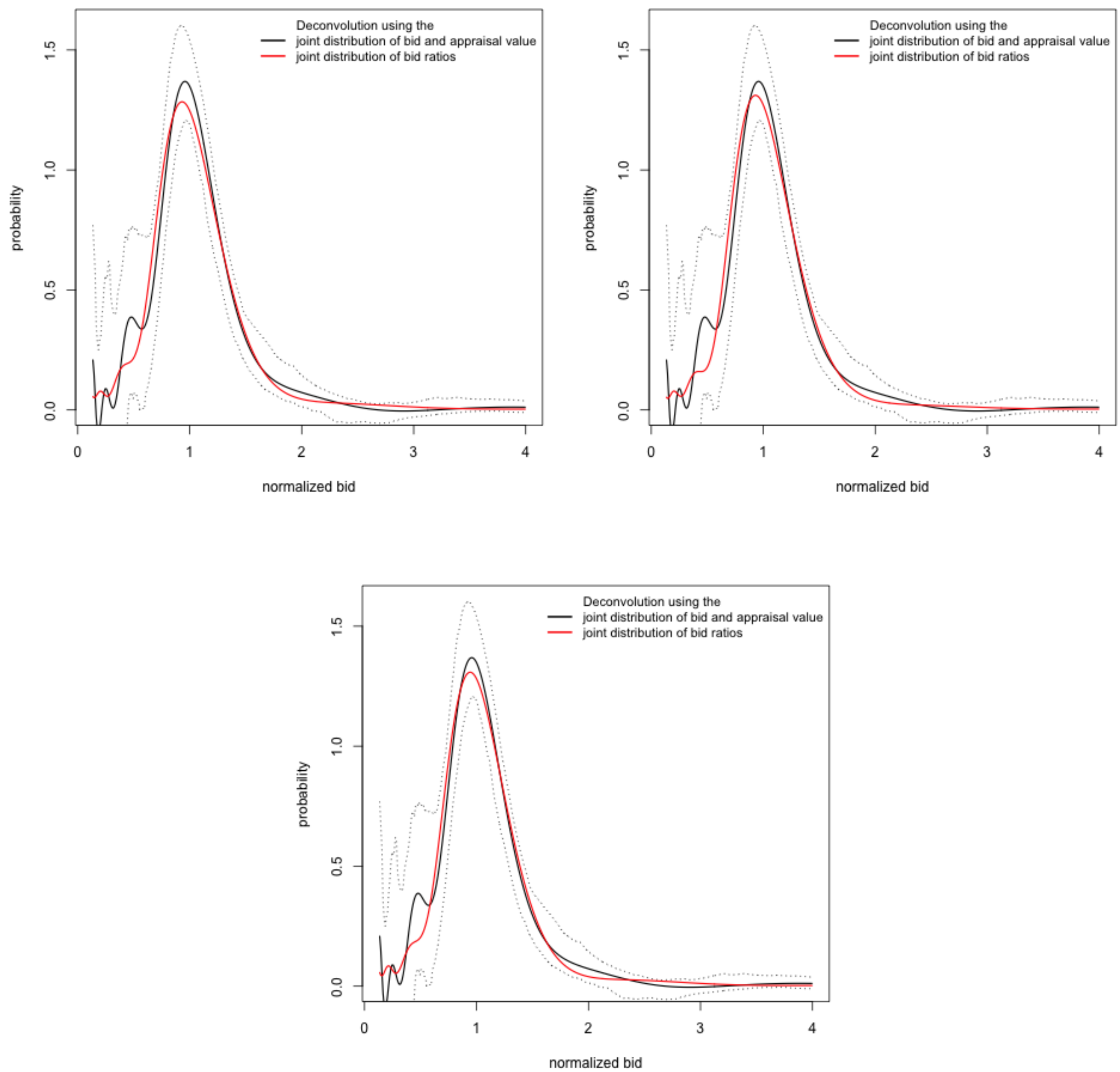

Figure 7: Estimated densities of the individual bid component using (1) the joint distribution of bid and appraisal value (one estimate), and (2) the joint distribution of two bids ratios (three estimates). The dotted lines show pointwise 95% confidence intervals for (1), estimated through a subsampling procedure.

seller's ex-ante appraisal value, (2) the mutual independence assumption between components entering bidders and seller's valuations.

7 Counterfactual analysis

In this section, the estimated distributions of values, seller information, observed and unobserved heterogeneity are combined to simulate a set of auctions under counterfactual informational structures and alternative reserve price policies. The outcomes of interest are the expected surplus and revenue per auction. We compare these outcomes to the seller current reserve price policy (i.e., a secret reserve price revised ex-post), denoted "baseline" hereafter. For the baseline policy, we use the estimated equilibrium bid function and revision rule.[39]

7.1 Simulation of counterfactuals

First-best outcome. We start by computing surplus and revenue under the assumption that the seller has perfect information about the unobserved auction heterogeneity component Y and announces a public reserve price equal to their true reservation value $X_0 \; Y$. Let R_0^{FB} denote this reserve price. Under symmetry of bidders' private values, the equilibrium bid function if $Y = 1$ has a simple closed-form expression

$$\beta(x) = x - \frac{1}{F_X(x)^{n-1}} \int_{r_0^{FB}}^{x} F_X(u)^{n-1} du.$$

This benchmark gives an upper bound on attainable surplus, because the auction is ex-post efficient. Additionally, the benchmark allow us to determine the benefit for the ONF of collecting more precise signals about unobserved tract heterogeneity.[40]

Alternative reserve price policies. We compare revenue and surplus under the current "baseline" policy to alternative reserve price rules, namely: (a) no reserve price, (b) a public reserve price, (c) secret reserve price, revised based on a convex combination of bids (mean) and appraisal value. These alternative policies are implemented as follows.

(a) Under symmetry of bidders' private values, the equilibrium bid function with no reserve price has a standard closed-form expression. Multiplicative separability of individual bid component and unobserved heterogeneity holds as in the model of Krasnokutskaya (2011).

(b) Denote by R_0^P the public reserve price announced by the seller. Due to bid shading in the first-price format, auction outcomes under a public and a (fixed) secret reserve

[39] Although the (binary) decision to accept the highest bid is non-parametrically identified, due to the curse of dimensionality we use the parametric specification shown in Table 5.

[40] We note that the first-best outcome is in fact implementable if the seller can gather reports about Y from each bidder in addition to their bids and incentivize them with large punishments if their answers do not coincide. In at least one equilibrium of this game, they will always report Y truthfully. The existence of efficient mechanisms in general auction settings was argued, for example, in McLean and Postlewaite (2004).

prices set both to R_0^P differ.[41] To restrict attention to the impact of the seller's learning from the bids, we assume that the public reserve price is set such that the allocation rule is identical to that generated by a fixed secret reserve price of $R_0 = X_0\widetilde{Y}$ (i.e., with no ex-post revisions). Details are included in Appendix D.1.[42]

(c) Under the last counterfactual policy, the ex-post reserve price is constructed as a convex combination of bids and appraisal value

$$R_1(X_0, \widetilde{Y}, \mathbf{b}; \gamma) = X_0\left(\gamma\overline{\mathbf{b}} + (1-\gamma)\widetilde{Y}\right) \tag{7}$$

where $\overline{\mathbf{b}}$ is the average bid received and $\gamma \in [0, 1]$. The function R_1 satisfies the homogeneity assumption (Assumption 1.1). Policy (c) coincides with policy (b) with a weight of 1 on the appraisal value. The interim expected profit of bidder i is given by

$$\pi(x_i, y; b_i) = (x_i y - b)P(b_i \geq R_1(X_0, \widetilde{Y}, b_i, \mathbf{B}_{-i}; \gamma) \cap b_i \geq B_j, j \neq i \mid Y = y) \tag{8}$$

A symmetric Bayes-Nash equilibrium is characterized by a function $\beta_y(.)$ such that $\pi(x_i, y; b_i)$ is maximized when $b_i = \beta_y(x_i)$, $b_j = \beta_y(x_j)$ for $j \neq i$ (for every $i \in \{1, ..., n\}$ and every realization of X_i). From the perspective of bidder i, the revision rule in Equation (7) depends on the equilibrium played through the distribution of $\mathbf{B}_{-i}$. An equilibrium is found numerically by best-response iteration. The details of this procedure are presented in Appendix D.2.

7.2 Counterfactual results

Figure 8 compares the equilibrium bid function under the current reserve price policy (baseline) to the equilibrium bid functions under policies (a), (b), and (c). In the latter, the cases with $\gamma \in \{0, 0.5, 1\}$ are plotted. The extreme cases (γ equals 0 or 1) provide upper and lower bounds for the equilibrium bid function under intermediate values of γ ($\gamma \in (0, 1)$).

The left panel shows that the equilibrium bid function under the current baseline policy (secret reserve price, ex-post revised) is close to the bid function with no reserve price, indicating that revisions relax any ex-ante commitment of the seller.[43] The right panel

[41] A secret reserve price excludes more bidder types than an otherwise identical public reserve price. Indeed, under a public reserve price of r, the item is allocated to the highest bidder with ***valuation*** greater or equal to r. Under a secret reserve price of r (with no ex-post revisions), the item is allocated to the highest bidder with ***bid*** greater or equal to r. Due to bid shading the latter marginal bidder must have a valuation strictly greater than r.

[42] The (binding) public reserve price is separable in Y, therefore, multiplicative separability of the equilibrium bid function still holds.

[43] A public reserve price clearly leads to more aggressive bidding than with no reserve price. A secret

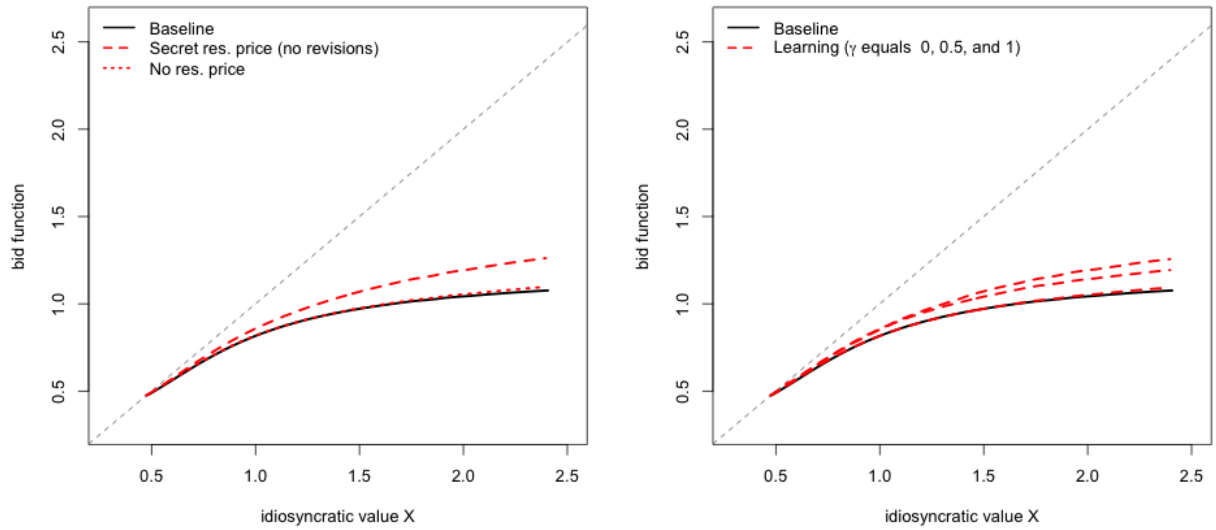

Figure 8: Equilibrium bid functions conditional on $Y = 1$ under the baseline and counterfactual reserve price policies. On the right panel, the topmost dashed function corresponds to $\gamma = 0$, while the bottom dashed function corresponds to $\gamma = 1$.

shows that as γ goes to 1, bidders bid less aggressively. This is consistent with the fact that as more weight is put on the average bid (in setting the ex-post reserve price), the highest bidder faces a lower ex-post reserve price.

Table 6 summarizes the results of the counterfactual analysis. The table records the expected surplus and revenue per auction under the current baseline and counterfactual reserve price policies. Average revenue and surplus per auction under the current policy are 13, 216€ and 17, 380€. Learning the realization of unobserved auction heterogeneity would allow the seller to increase revenue by 5.77% and surplus by 5.22% (first-best outcome).[44] Announcing a public reserve price increases revenue by 5.86% and reduces surplus by 2.34%. Without a reserve price, surplus would increase by 3.35% whereas revenue decreases by only 1.46%. When employing a reserve price, the fraction of tracts sold is highest under the first-best, followed by the baseline reserve price policy and finally the public reserve price policy.

reserve price that *depends* on the bids, on the other hand, has an ambiguous effect on equilibrium bidding relative to the no reserve price case. See Appendix C for a characterization of equilibrium bidding strategies.

[44]It is worth emphasizing that both surplus and revenue increase in the first-best outcome. Surplus increases because we assume the seller has full information and can set the public ex-post efficient reserve price. Revenue increases because the seller announces a public reserve price (i.e., there are no revisions) that induces more aggressive bidding.

	Baseline	Public Res. Price	No Res. Price	First-Best
Revenue per auction (mean, in 2003 €)	13,216	13,991	12,919	13,961
95% CI	[12,029, 14,392]	[12,639, 15,335]	[11,721, 14,221]	[12,733, 15,224]
Change relative to baseline (in %)	0	5.86	-1.46	5.77
Surplus per auction (mean, in 2003 €)	17,380	16,984	17,850	18,164
95% CI	[14,810, 20,517]	[14,392, 19,989]	[15,124, 21,367]	[15,482, 21,415]
Change relative to baseline (in %)	0	-2.34	3.35	5.22
Fraction of tracts sold (mean)	0.77	0.61	1	0.85

Table 6: This table shows counterfactual outcomes under the different reserve price policies and the first-price auction format. The "baseline" correspond to the current reserve price policy used by the ONF. The first-best outcome corresponds to a situation where the ONF faces no uncertainty about Y and sets the ex-post efficient public reserve price.

Figure 9 shows (percentage) change in surplus and revenue relative to the baseline for the counterfactual reserve price policies. In particular, we plot the change in outcomes from the adoption of the ex-post reserve price given by policy (c) for different values of $\gamma \in [0, 1]$.

When γ equals zero, the ex-post reserve price yields the same outcomes as a public or secret reserve price (with no revisions).[45] As γ goes to one, the ex-post reserve price incorporates information from the bids to update estimates of Y. Allocative efficiency increases. Since bidders face a lower reserve price (in expectation), bidding is less aggressive and revenue decreases. There is an interior value of the parameter ($\gamma = 0.87$) that maximizes surplus per auction. Under this value, the seller efficiently combines information from their appraisal value and the bids (within the class of learning rules given by (c)).

The surplus gap between the public reserve price and the first-best is reduced by 84% by the adoption of a secret reserve price policy with efficient learning ($\gamma = 0.87$). Under efficient learning, surplus increases by 5.84% and revenue decreases by 4.92% compared to outcomes under a public reserve price. The seller trades off greater allocative efficiency against lower revenue per auction.

7.3 Robustness checks

Dynamics. Tracts are auctioned sequentially within a sale and the ten sales are held sequentially over two months. We consider, therefore, whether there are any dynamic effects within and across sales given that our structural model is static and assumes independence across auctions and sales. To choose the ordering of tracts within a sale, the ONF picks the first tract randomly and starting from it, tracts are auctioned by alphabetical order.

[45] Recall that the public reserve price is set to generate the same allocation rule as a (fixed) secret reserve price of R_0.

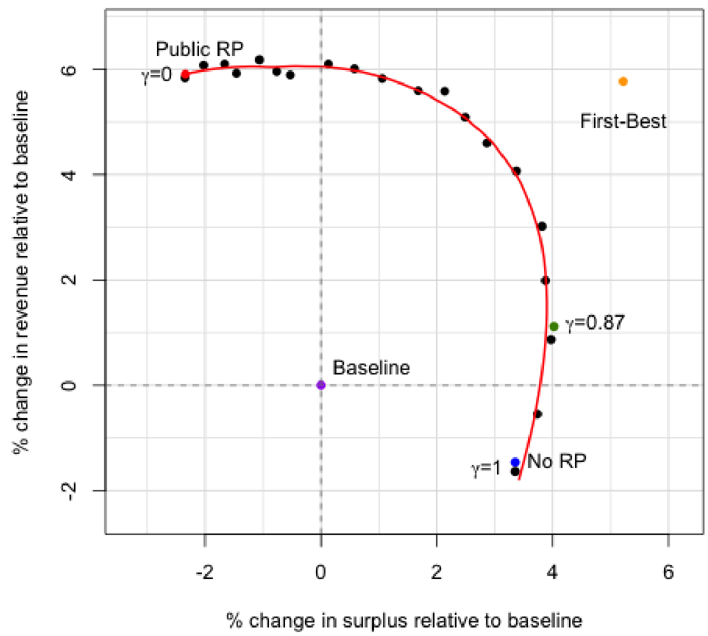

Figure 9: Counterfactual (expected) changes in surplus and revenue per auction from adoption of alternative reserve price policies. The baseline corresponds to the current policy used by the ONF. The public reserve price corresponds to the case where the ex-ante reserve price is announced (and not revised). The first-best corresponds to the full information case. Black dots corresponds to an ex-post reserve price (learning) $R_1(X_0, \widetilde{Y}, \mathbf{b}; \gamma)$ given by Equation (7) where γ in $[0, 1]$ for 20 points on the interval. The red curve is a smoothed fit of this frontier.

Within-sale dynamics can be assessed by examining whether tract order affect firms' bidding behavior. We regress the (log) average bid per auction on observed tract characteristics and tract order (in the sale), for each sale separately. Figure 10 (left panel) shows the coefficient estimate of the tract order. Except for sales $\{8, 10\}$, tract order is not correlated with bids. Excluding these sales from the analysis does not qualitatively affect our counterfactual results.

Across sale dynamics are assessed by examining sale dummies in the regression of bids on tract characteristics (all sales are pooled). Figure 10 (right panel) shows the difference in the (log) average bid per auction between sale $i \neq 6$ and the reference sale (sixth), controlling for tract characteristics. Notably, bids are on average higher in the first two sales of the bidding season. Restricting our sample to later sales or intermediate sales (e.g., 3 to 8) does not qualitatively affect our results.

Endogenous participation. Next, we discuss how bidders' participation may affect our results.

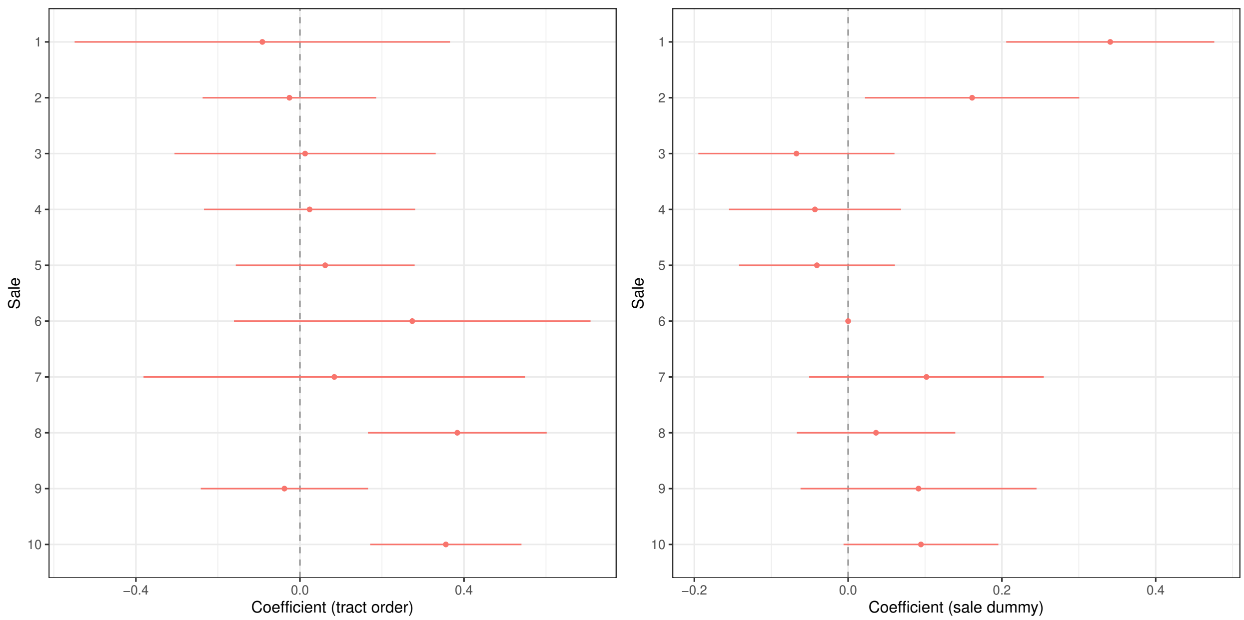

Figure 10: The left panel shows the effect of tract order (within sale) on the (log) average bid submitted, controlling for observed auction heterogeneity. The right panel shows estimates of sale dummies in the regression of (log) average bid on observed auction heterogeneity. The omitted category is the sixth sale.

In our setting, a bidder "enters" a given auction when they cruise the auctioned tract (cruises account for most of entry costs).[46] Before making their entry decision, potential bidders have access to observable tract characteristics reported in the sale booklet and private signals correlated with their private valuations for the tract. After entering, bidders observe the unobserved auction heterogeneity component and their private valuation.[47]

Endogenous participation can affect our counterfactual predictions (in particular, regarding the effect of announcing a public reserve price) if a reserve price is announced *before* bidders make their entry decisions (e.g., the reserve price is reported in the sale booklet). The ex-ante effect on surplus and revenue is, however, ambiguous. Announcing a public reserve price may increase participation for tracts with below-average (seller) reservation value, but reduce participation for tracts with above-average (seller) reservation value.

If the reserve price is publicly announced *only after* bidders make their entry decisions (e.g., the day of the sale), then allowing for endogenous entry does not alter our counterfactual predictions. Indeed, with such a disclosure policy, bidders condition on the same

[46] According to industry professionals, firms which cruise a tract typically bid in the auction. Although we cannot test this assumption using our data, Athey and Levin (2001) also report similar evidence for the U.S. Forest Service auctions.

[47] Grundl and Zhu (2018) show that, in this setting, (i) the value distribution of entrants conditional on Y is independent of the number of entrants; (2) if bidders do not observe Y before entering, then separability of Y and the private valuation of potential bidders carries to entrants.

information when making their entry decision (i.e., tract characteristics reported in the sale booklet and some private signals about their valuations) as under the secret reserve price policy we consider.

Asymmetries between firms. The estimated model assumes bidders are symmetric. We investigate whether our results are robust to this assumption. In the auction data set, each firm is identified by a bidder code which allows us to compare participation rates across bidders.[48] Bidders participate on average in 27 auctions over the sale season (with a standard deviation of 37).

To identify bidder types, we group bidders by decile of the distribution of participation. Then, bids are regressed on tract characteristics and dummies for each decile (i.e., ten dummies for whether bidder i is in decile $k \in \{1, \ldots, 10\}$ of the distribution of participation). The results are shown in the left panel of Figure 11. We identify two types of bidders: regular bidders corresponding to the top two deciles of the participation distribution and fringe bidders. There are 43 regular bidders who participate on average in 84 auctions per bidder, and 163 fringe bidders who participate on average in 11 auctions per bidder.

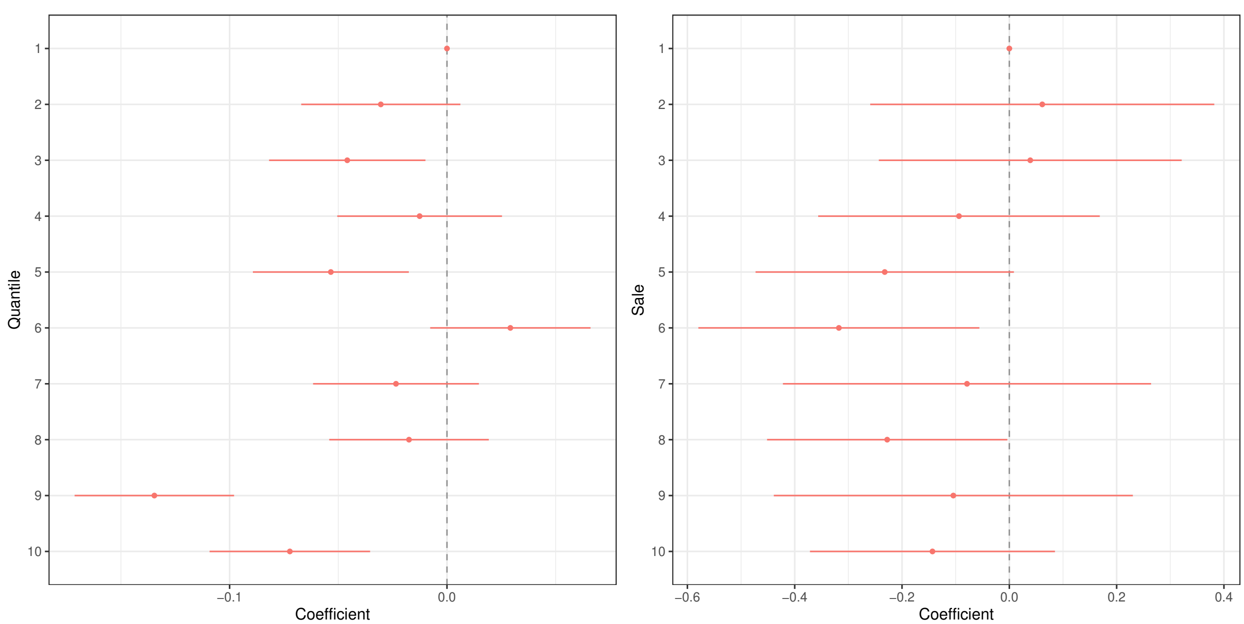

Figure 11: The left panel shows the coefficients on the dummies for each decile of the participation distribution in the regression of average bid on observed auction heterogeneity. The right panel shows estimates of sale dummies in the probit regression of the ONF's decision to revise the reserve price down on observed auction heterogeneity. The omitted category is the first sale.

[48]Firm names, which would allow to recover other bidder-specific covariates such as mills' locations, are unfortunately not disclosed by the ONF.

The model is estimated allowing the distribution of private values X_i to differ across regular and fringe bidders. We focus on three-bidder auctions with two regular and one fringe bidder. The estimation approach is modified to account for different first-order conditions across bidder types in step 4.[49]

The results are shown in Figure 12. The top left panel shows the empirical cumulative distribution of bids (in logarithm, residualized following step 1). Regular bidders tend to bid slightly more aggressively (i.e., higher) than fringe bidders but the difference is relatively small. The top right and bottom panels show the estimated densities of normalized bids (recovered from the deconvolution method) and private values for regular and fringe bidders. We find that the equilibrium bid functions (in auctions with two regular and one fringe bidder) differ by at most 1%. Taken together, these findings indicate that although firms differ in their participation decisions, this does not translate into significant variation in private valuations and equilibrium bidding patterns.

Revenue constraints. The ONF must cover its operating costs using proceeds from timber auctions. Therefore, instances where the ONF revises its ex-ante reserve price down may be reflecting revenue constraints rather than learning from the bids about unobserved tract characteristics.[50] While discussions with ONF officers indicate that these revenue constraints are in general not binding, we examine whether revision decisions differ across the ten sales over the bidding season. If revenue constraints were important, we might expect the ONF to revise down more often near the end of the sale season (to achieve budget balance). The results are shown in the right panel of Figure 11. Except in the sixth sale in which the ONF was less likely to revise its reserve price, revision decisions appear fairly similar across sales.

Variation in the number of bidders. Table 3 shows that the number of bidders per auction is correlated with some of the tract characteristics. Given Assumption 2, this correlation might call into question the assumption of no selection of bids based on observables (i.e., full participation). We investigate whether this pattern can be explained by variation in the number of active firms across different locations of the Grand Est region; and these locations being correlated with tract-level observed characteristics (e.g., grapeshot damage).

Although we do not observe the exact location of mills, we do have information on the forest to which each tract belongs. There are 789 different forests in our sample, with on average 2.88 tracts per forest. The regression of the (log) number of participants on forest fixed effects yields an (adjusted) R^2 of 41%. Including tract-level covariates, in addition to

[49]In implementing the deconvolution (step 2 in the estimation approach), we use the joint distribution of a regular and fringe bid per auction and impose the normalization $E[\log(A_1)] = 0$ for fringe bidders.

[50]The constraint can stipulate, for instance, that total revenue over all auctioned tracts must be greater or equal to the ONF's operating costs for that year.

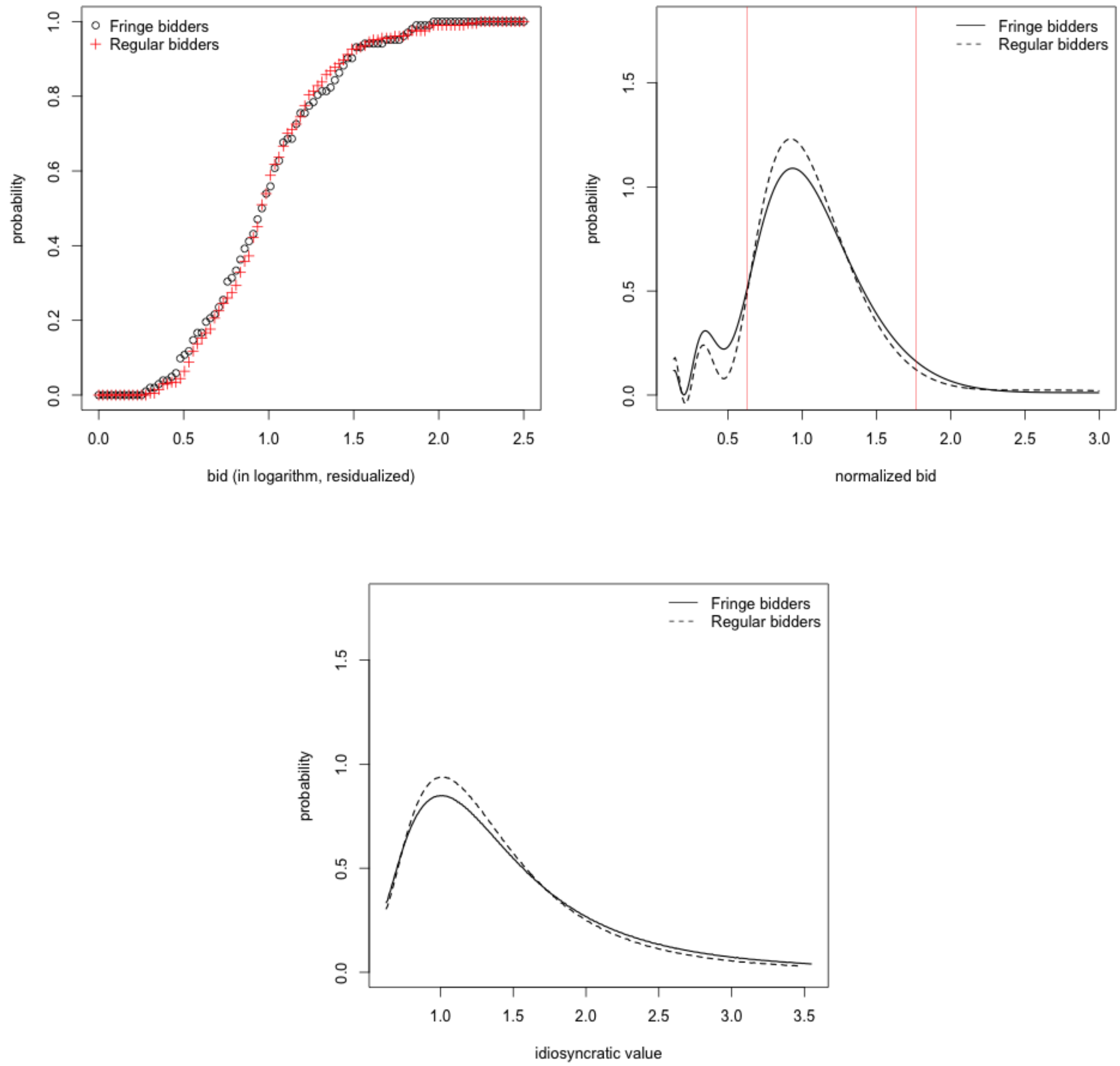

Figure 12: Empirical cumulative distribution of bids (top left), the estimated densities of normalized bids (top right), and the estimated densities of private values (bottom) for regular and fringe bidders. The red vertical lines represents the estimates of support boundaries.

the forest effects, increases the R^2 to 54%. This indicates that the spatial distribution of tracts relative to firms explain a significant fraction of participation decision; although we cannot completely rule out covariates' explanatory power (the effects of tract characteristics are however smaller and/or insignificant once forest FE are included).

As a robustness check, we conduct our estimation and counterfactual exercises including forest fixed effects in the first-step homogeneization of bids (the fixed effects are excluded from the residualized bids). The sample used contains all tracts belonging to a forest with more than one auctionned tract. The results are quantitatively similar despite relying on the smaller sample size.

8 Conclusion

The common use of secret reserve prices in auction markets has been a persistent empirical puzzle. This paper investigates a novel rationale for their use. If the seller is less informed than the bidders about the underlying heterogeneity of the auctioned item, she may learn from the bids and adjust her initial appraisal value. Doing so allows the seller to allocate the item more efficiently, albeit at a cost of lower revenue.

The French timber industry provides an empirical setting where this type of information asymmetry between bidders and seller is important. Additionally, the ONF uses a secret reserve price which can be revised down if no bid is above it. We build a model of bidding in first-price auctions that captures these two features and show that the model is identified from data on bids, allocation rule, and the ONF's appraisal value.

Using the estimated model, we conduct counterfactual analysis of alternative reserve price policies. The results show that the seller would benefit substantially from acquiring more precise signals about unobserved auction heterogeneity. Using the information conveyed by bids allows the seller to improve allocative efficiency. However, learning from the bids induces less aggressive bidding and lower revenue by relaxing the seller's commitment power (or conversely increasing bidders' market power). From a broader perspective, the results speak to the importance of a seller's appraisal technology in auction markets. In the context of the ONF's timber auctions, the seller uses the information revealed by the bids to supplement their imperfect appraisal of tracts.

Finally, we note possible extensions to the current framework. Alternative approaches for dealing with unobserved heterogeneity can be used. For instance, one can relax the separability assumption between unobserved heterogeneity and private value components, if the former is assumed to be discrete. The misclassification approach of Hu et al. (2013) can then be employed (e.g., using a triplet formed of two bids and the appraisal value for a given auction). Our identification strategy (Step 2, in particular) can also be applied to other non-standard auction rules. For instance, in Arozamena and Weinschelbaum (2009) and Chiang and Sa-Aadu (2014), a favored player can take action upon seeing other players bids (e.g., right of first refusal). The event of being out-bid by a favored player is to some extent similar to how the ex-post reserve price acts in our model, and the equilibrium strategies may be identified in a similar fashion.

A Proofs

A.1 Proof of Lemma 1 and Remark 1

Lemma 1. Under Assumption 1, there exist unique functions $R_0(x_0, \tilde{y})$ and $R_{-i1}(x_0, \tilde{y}, \mathbf{b}_{-i})$ defined as solutions to $R_1(x_0, \tilde{y}, R_0, \ldots, R_0) = R_0$ and $R_1(x_0, \tilde{y}, \mathbf{b})|_{b_i=R_{-i1}} = R_{-i1}$. These functions satisfy

1. $b_i > R_0(x_0, \widetilde{y}),\ b_i > \max_{j \neq i} b_j \implies b_i > R_1(x_0, \widetilde{y}, \mathbf{b})$
2. $b_i > R_1(x_0, \widetilde{y}, \mathbf{b}) \iff b_i > R_{-i1}(x_0, \widetilde{y}, \mathbf{b}_{-i})$

for all i and $(x_0, \tilde{y}, \mathbf{b}_{-i})$. Moreover, R_{-i1} is continuous in all arguments and homogeneous of degree 1 in bids and appraisal value.

Proof. Homogeneity of degree 1 implies, via differentiation with respect to λ, that

$$\sum_i \frac{\partial \log R_1}{\partial \log b_i} + \frac{\partial \log R_1}{\partial \log \widetilde{y}} = 1 \tag{9}$$

To prove the existence and uniqueness of R_0, we fix $x_0, \tilde{y}$ and define the function

$$h(z) = \log R_1(x_0, \tilde{y}, b, \ldots, b)|_{b=e^z}$$

on the real line. Note that, by definition, R_0 is equal to the exponential of the solution to $h(z) = z$. We therefore need to show that there exists a unique solution to this equation on the real line.

From Equation (9), the total derivative of $\log R_1(x_0, \widetilde{y}, b, \ldots, b)$ with respect to $\log b$ is bounded

$$\frac{\partial h(z)}{\partial z} = \frac{d \log R_1(x_0, \widetilde{y}, b, \ldots, b)}{d \log b}\Big|_{b=e^z} = 1 - \frac{\partial \log R_1(x_0, \widetilde{y}, b, \ldots, b)}{\partial \log \widetilde{y}}\Big|_{b=e^z} < 1,$$

because R_1 is strictly increasing in $\tilde{y}$ and both R_1 and $\tilde{y}$ are positive. Note that, since R_1 is continuous and positive, there exist $\underline{z}$ such that $h(\underline{z}) > \underline{z}$. Moreover, by Assumption 1.2 there exist $\overline{z}$ such that $h(\overline{z}) < \overline{z}$. Therefore, $h(z)$ crosses the 45 degree line at least once in the range $[\underline{z}, \overline{z}]$ by the Intermediate Value Theorem. Finally, because $\frac{\partial h(z)}{\partial z} < 1$, the crossing is unique.

To prove the first implication, observe that for any value z to the right of the crossing,

$h(z)$ is below the 45 degree line. Therefore, setting $z = b_{(1)} := \max_j b_j$, we have

$$
\begin{gathered}
R_0(x_0, \tilde{y}) < b_{(1)} \quad \Rightarrow \quad \log R_0(x_0, \tilde{y}) < \log b_{(1)} \quad \Rightarrow \\
\log R_1(x_0, \tilde{y}, e^{\log b_{(1)}}, \ldots, e^{\log b_{(1)}}) = h(\log b_{(1)}) < \log b_{(1)} \quad \Rightarrow \\
R_1(x_0, \tilde{y}, b_{(1)}, \ldots, b_{(1)}) < b_{(1)} \quad \Rightarrow \quad R_1(x_0, \tilde{y}, \mathbf{b}) < b_{(1)},
\end{gathered}
$$

by strict monotonicity of R_1. That is, $b_{(1)} > R_0(x_0, \tilde{y})$ implies $b_{(1)} > R_1(x_0, \tilde{y}, \mathbf{b})$ for all possible $\mathbf{b}$, such that $b_{(1)} = \max_j b_j$.

To prove the existence of R_{-i1}, we fix $x_0, \tilde{y}, \mathbf{b}_{-i}$ and define the function

$$
g(z) = \log R_1(x_0, \tilde{y}, \mathbf{b}_{-i}, b_i)|_{b_i = e^z}
$$

on the real line. By definition, R_{-i1} is the exponential of the solution to $g(z) = z$. We therefore need to show that there exists a unique solution to this equation on the real line.

Using the same arguments as for $h(z)$, we can show that the function $g(z)$ crosses the 45 degree line at least once. It only remains to show that $\frac{\partial g(z)}{\partial z} < 1$. However, Equation (9) yields

$$
\begin{gathered}
\frac{\partial g(z)}{\partial z} = \frac{d \log R_1(x_0, \widetilde{y}, \mathbf{b}_{-i}, b_i)}{d \log b_i}\Big|_{b_i = e^z} = \\
1 - \sum_{j \neq i} \frac{\partial \log R_1(x_0, \widetilde{y}, \mathbf{b}_{-i}, b_i)}{\partial \log b_j}\Big|_{b_i = e^z} - \frac{\partial \log R_1(x_0, \widetilde{y}, \mathbf{b}_{-i}, b_i)}{\partial \log \widetilde{y}}\Big|_{b_i = \log z} < 1,
\end{gathered}
$$

because R_1 is strictly increasing in $\tilde{y}, \mathbf{b}_{-i}$ and $R_1, \tilde{y}, \mathbf{b}_{-i}$ are all positive.

The second implication follows from the fact that $g(z)$ crosses the 45 degree line exactly at $z = \log R_{-i1}(x_0, \tilde{y}, \mathbf{b}_{-i})$. To show continuity of R_{-i1} and R_0, note that they are unique minimizers of $(g(z) - z)^2$ and $(h(z) - z)^2$ respectively, and thus, are continuous in all arguments by the Maximum Theorem.

Finally, by homogeneity of degree 1 of R_1

$$
\begin{gathered}
b_i > R_{-i1}(x_0, \widetilde{y}, \mathbf{b}_{-i}) \quad \Leftrightarrow \quad b_i > R_1(x_0, \widetilde{y}, \mathbf{b}) \quad \Leftrightarrow \\
b_i > R_1(x_0, \lambda\widetilde{y}, \lambda\mathbf{b})/\lambda \quad \Leftrightarrow \quad b_i > R_{-i1}(x_0, \lambda\widetilde{y}, \lambda\mathbf{b}_{-i})/\lambda.
\end{gathered}
$$

for all $\lambda > 0$. Therefore, $R_{-i1}(x_0, \widetilde{y}, \mathbf{b}_{-i}) = R_{-i1}(x_0, \lambda\widetilde{y}, \lambda\mathbf{b}_{-i})/\lambda$, that is, R_{-i1} is homogeneous of degree 1 in bids and appraisal value. □

Remark 1. *Under Assumption 1, $R_0 = \tilde{Y}/h(X_0)$ for some function $h(.)$.*

Proof. From the definition of R_0 and invoking homogeneity of degree 1, we have

$$1 = R_1(X_0, \frac{\tilde{Y}}{R_0}, 1, \ldots, 1).$$

Since the ex-post reserve price R_1 is strictly increasing in the appraisal value, by the Inverse Function Theorem, there exist a function h such that $\frac{\tilde{Y}}{R_0} = h(X_0)$ locally, and therefore also globally on a compact $[\underline{x}_0, \overline{x}_0]$. $\square$

A.2 Proof of Proposition 1

Proposition 1 Under Assumption 1, if $\alpha(.)$ is an equilibrium bidding strategy of the game indexed by $y = 1$, then an equilibrium bidding strategy in the game indexed by y, with $y \in [\underline{y}, \overline{y}]$, is such that $\beta_y(x_i) = y\alpha(x_i)$, for all i, and $\alpha(\underline{x}) = \underline{x}$. Moreover, all types with private value above $\underline{x}$ have a strictly positive probability of winning.

Proof. Let all but players but i follow the same bidding strategy $\beta_y(.) = y\alpha(.)$. Consider an auction indexed by $Y = y$, then the interim expected profit of bidder i, given a bid b_i, is

$$\pi(x_i, y; b_i) = (yx_i - b_i)P(b_i \geq Z_y \cap b_i \geq yA_j, j \neq i | Y = y)$$

where $Z_y = R_{-i1}(X_0, yS, y\mathbf{A}_{-i})$. By homogeneity of degree one of R_{-i1}, we have that $Z_y = yZ_1$, therefore

$$\begin{aligned}\pi(x_i, y; b_i) &= y(x_i - b_i/y)P(b_i/y \geq Z_1 \cap b_i/y \geq A_j, j \neq i), \\ &= y \cdot \pi(x_i, 1; b_i/y)\end{aligned}$$

Clearly, if $\beta_1(x_i) = \alpha(x_i)$ maximizes $\pi(x_i, 1; b_i)$, then $\beta_y(x_i) = y\beta_1(x_i)$ also maximizes $\pi(x_i, y; b_i)$, which proves that $\beta_y(x_i) = y\alpha(x_i)$.

To address the participation and boundary condition, we assume first that all types participate. Note that in a symmetric equilibrium, it cannot be that $\alpha(\underline{x}) < \underline{x}$ or the lowest type would have an incentive to increase its bid (deviate upwards). In addition, it cannot be that $\alpha(\underline{x}) > \underline{x}$ or the types in the range $(\underline{x}, \alpha(\underline{x}))$ would have a strictly positive probability of winning with strictly negative profits.

Consider now the situation where a positive measure of types choses not to participate. Then there is a positive probability that there are no participants in the auction. Since we model that the seller replaces missing bids with the minimum (among existing) bid, the non-participating agent with any type $x > \underline{x}$ has a positive probability of facing no opponents, if he chooses to participate. Moreover, if he submits $\underline{x}$, he will have a strictly positive probability of winning against the ex-post reserve price, as

$$R_1(x_0, \tilde{y}, \underline{x}, \ldots, \underline{x}) < \underline{x} \leqslant x$$

by Assumption 1.3. Therefore, there is full participation.

Finally, to show that the probability of winning, given normalized bid a, is strictly positive for all $a > \underline{x}$, recall first that the probability of winning is non-decreasing. We therefore only have to show that this probability is strictly increasing in the neighborhood of $\underline{x}$.

By Assumption 1.3, there is a neighborhood of $\underline{x}_0, \underline{s}, \underline{x}$ such that for all x_0, s, x in that neighborhood $R_1(x_0, s, x, \ldots, x) < x$. Therefore, by continuity of the bidding strategy, for a normalized bid close to $\underline{x}$, the probability of winning against rival bids and the ex-post reserve price is strictly positive.□

A.3 Proof of Lemma 2

Lemma 2. Under Assumption 1, the event $\widetilde{\omega_i}$ is equal to the event ω_i. The distribution of M is identified from the deconvolution of the distributions of L and Y.

Proof. The first result is a direct implication of homogeneity of degree 1 of R_1 in bids and appraisal value.

For the second result, note that

$$\begin{aligned} P(B_i \leq b | \widetilde{\omega_i}) &= \int P(A_i Y \leq b | \widetilde{\omega_i}, Y = y) f_Y(y | \widetilde{\omega_i}) dy \\ &= \int P(A_i \leq \frac{b}{y} | \widetilde{\omega_i}, Y = y) f_Y(y) dy \\ &= \int P(A_i \leq \frac{b}{y} | \widetilde{\omega_i}) f_Y(y) dy \end{aligned} \tag{10}$$

where the second equality is a consequence of Y being independent of $\widetilde{\omega_i}$, and the third equality is a consequence of A_i being independent of Y conditional on the event $\widetilde{\omega_i}$.Therefore, the random variable L is the convolution of the random variables M and Y. The latter variables are independent by construction. □

B Institutional details

B.1 Tract characteristics

This section provides a more detailed description of the categorical variables presented in Table 2.

The variable "Stand" corresponds to the forest type, which can either be a high forest, i.e., a forest originating from seed or from planted seedlings and consisting of large, tall mature trees with a closed canopy; or a coppice forest, i.e., a forest produced from vegetative regeneration. A coppice forest can be converted into a high forest ("Conversion of a stand"). A hybrid of high and coppice forests can also be managed ("Coppice with standards").

The variable "Cut" corresponds to the type of cutting required, which can take many forms: selection cutting eliminates weaker or low value trees to make space for the remaining trees, regeneration cutting consists of harvesting older trees to allow the growth of younger age classes, etc.

The variable "Grapeshot" corresponds to the level of grapeshot damage from artillery used in WWI. "Landing area" indicates whether a designated space is available for storing the harvested timber before transporting it to the mill. "Conditions" describes the difficulty in logging (or cutting) the trees and skidding (or extracting), the process by which the cut tree is moved through the forest to the landing area.

B.2 Role of the ex-ante reserve price

In our empirical application, the seller sets an *ex-ante* reserve price R_0, that is observed in the data. Note, however, that from the perspective of the bidders, only the ex-post reserve price R_1 is relevant to their bidding behavior. Indeed, bidder i's interim profits are given by

$$(x_i y - b_i) P(b_i \geq R_1(X_0, \widetilde{Y}, b_i, \mathbf{B}_{-i}) \cap b_i \geq B_j, j \neq i \mid Y = y)$$

Why does the seller define an ex-ante reserve price if it is irrelevant to auction outcomes? This section provides several justifications for the use of the ex-ante reserve price based on our discussions with ONF officers.

The allocation rule (see Figure 2) reveals that tracts are always sold when the highest bid exceeds the ex-ante reserve price. Arguably, many combinations of R_0 and R_1 can generate this pattern: for instance, the seller could set R_0 to an arbitrarily low value (or not define it at all) and revise more often (via an alternative rule R_1) to achieve the exact same allocation observed in Figure 2. Evidence gathered through discussion with the auctioneer indicate, however, that the ONF defines an ex-ante reserve price to simplify the revision decision of the ONF officer conducting the auction. By using the simple rule "accept the maximum bid if it is higher than R_0," the ONF minimizes the number of instances in which the auctioneer must incorporate information from the bids and revise their appraisal value (which can be computationally costly and time consuming). Moreover, the ex-ante reserve price serves as a reference point that limits, to some degree, the discretion of the ONF officer conducting the auction.

Second, as described in Section 3, if all bids are rejected, the ONF announces its ex-ante reserve price. Therefore, setting R_0 before the auction to a sensible value allows the ONF to reveal its reservation value to the bidders, demonstrating its willingness to sell the tract (rather than collecting bids without intending to sell, simply to learn about bidders' willingness to pay, as in the setting studied by Olimov (2013)). Finally, defining an ex-ante reserve price before the auction also serves accounting purposes as it allows the ONF to compute an ex-ante estimate of the aggregate proceeds they should expect from the sale.

C Characterization of equilibrium bidding strategies

In our model, the equilibrium bidding strategy does not have a closed-form solution, because the ex-post reserve price depends on the bids, and thus, implicitly, on the endogenous bidding strategy. Therefore, a constructive characterization of an equilibrium bidding strategy is out of reach. This section proposes two alternative characterizations of the equilibrium bidding strategy.

Note that in an auction indexed by $Y = 1$, the equilibrium bidding strategy solves the following optimization problem

$$\beta_1(x) \in \arg\max_b (x - b)\tilde{W}_1(b),$$

where $\tilde{W}_1(b)$ is the equilibrium probability of winning against all other opponents and the ex-post reserve price. By the Envelope Theorem, the slope of the optimized interim bidder's profits at type x is equal to the equilibrium probability of winning $\tilde{W}_1(b)$ at the optimum bid $b = \beta_1(x)$. Recalling that $\beta_1(\underline{x}) = \underline{x}$, we can write the envelope condition as

$$\beta_1(x) = x - \frac{\int_{\underline{x}}^{x} \tilde{W}_1(\beta_1(z))dz}{\tilde{W}_1(\beta_1(x))}. \tag{11}$$

Without the ex-post reserve price, $\tilde{W}_1(\beta_1(x))$ would be equal to the distribution of $\max\{\mathbf{X}_{-i}\}$, that is, $F_X^{n-1}(x)$. In our application, $\tilde{W}_1(\beta_1(x))$ depends on the strategies of other players. Precisely, it is equal to the distribution of $\max\{\beta_1^{-1}(R_{-i1}(X_0, S, \mathbf{A}_{-i})), \mathbf{X}_{-i}\}$, which cannot be further simplified. Therefore, $\beta_1(x)$ corresponds to a fixed point in the space of strategies, satisfying (11).

An alternative characterisation can be obtained via first order conditions. We decompose $\tilde{W}_1(b)$ into the product of $W_1(b)$ - the equilibrium probability of winning against all other opponents and the ex post reserve price, conditional on being the highest bidder, and $F_X^{n-1}(\beta_1^{-1}(b))$ - the probability of being the highest bidder. Since $W_1(b)$ is non-decreasing by Lemma 1, the profit has increasing differences in (x, b), that is, the single-crossing property is satisfied. Therefore, the solution to the first-order conditions also satisfies the second-order conditions. The strategy $\beta_1(x)$ solves the following ordinary differential equation (ODE)

$$\beta_1'(x) = \frac{(x - \beta_1(x))(n-1)f_X(x)/F_X(x)}{1 - (x - \beta_1(x))w_1(\beta_1(x))/W_1(\beta_1(x))}, \quad \beta_1(\underline{x}) = \underline{x}, \tag{12}$$

where $w_1(b)$ is the derivative of $W_1(b)$. When the secret reserve price is independent of bids, $W_1(b)$ is a known distribution, and thus an equilibrium can be found by numerically solving the ODE.

Finally, denoting $\xi_1(x)$ the inverse bidding strategy, $\xi_1(x)$ solves the following ODE

$$\xi_1'(b) = \frac{F_X(\xi_1(b))}{(n-1)f_X(\xi_1(b))}\Big[\underbrace{\frac{1}{(\xi_1(b) - b)}}_{\text{standard auction}} - \underbrace{\frac{w_1(b)}{W_1(b)}}_{\text{secret r.p.}}\Big], \quad \xi_1(\underline{x}) = \underline{x}. \tag{13}$$

The latter expression shows that the effect of the secret reserve price on the bidding strategy is ambiguous. The direction in which the strategy shifts depends on the sign of $w_1(b)$, which can be positive or negative, because $W_1(b)$ is not necessarily a c.d.f (as it is the ratio of $\tilde{W}_1(b)$ over $F_X^{n-1}(\beta_1^{-1}(b))$). If the secret reserve price is independent of bids, $W_1(b)$ is a c.d.f, and thus the effect of a secret reserve price on the equilibrium bidding strategy would be to induce more aggressive bidding than with no reserve price.

D Counterfactual Analysis

D.1 Counterfactual public reserve price

We derive in this section the expression for the counterfactual public reserve price that yields the same allocation rule as a fixed secret reserve price of $R_0 = X_0\widetilde{Y}$.

Let $\beta_y^s(.)$ denote a symmetric Bayesian Nash equilibrium of a first-price auction (indexed by $Y = y$) with a *fixed* secret reserve price equal to $R_0 = X_0\widetilde{Y}$. Under this reserve price policy rule, we assume that the seller commits not to revise her reserve price after observing the bids. The reserve price is separable in Y (recall that $\widetilde{Y} = S - Y$). An argument similar to Proposition 1 shows that $\beta_y^s(.)$ is also separable in Y. We have

$$\beta_y^s(x) = y\beta_1^s(x)\ , \quad \forall x \in [\underline{x}, \overline{x}]$$

Let ξ_y^s denote the inverse bid function corresponding to $\beta_y^s(.)$. Define the public reserve price

$$R_0^P = Y \quad \xi_1^s \left(\frac{R_0}{Y}\right)$$

Lemma 3. *A public reserve price equal to R_0^P generates the same allocation rule as a secret reserve price equal to R_0.*

Proof. The allocation rule under a secret reserve price equal to R_0 is given by

$$q^s(\mathbf{x}, y) = \begin{cases} 1 & \text{if} \max_i \beta_y^s(x_i) \quad R_0 \\ 0 & \text{otherwise} \end{cases}$$

where $\mathbf{x}$ is the vector of bidders' private values. The allocation rule under a public reserve price equal to R_0^P is given by

$$q^P(\mathbf{x}, y) = \begin{cases} 1 & \text{if} \max_i y x_i \quad R_0^P \\ 0 & \text{otherwise} \end{cases}$$

From the definition of R_0^P, the two allocation rules are identical, and generate, by the revenue equivalence theorem, the same expected auction outcomes. □

We note two important considerations. In practice, the public reserve price R_0^P cannot be implemented because it is a function of the unobserved auction heterogeneity component Y. This component is not observed by the seller. In our counterfactual exercise, we simulate each auction by drawing from the model primitives (e.g., Y) and can therefore construct the public reserve price; or equivalently, we can use the fixed secret reserve price R_0, generating the same allocation.

Second, the equilibrium bid function β_y^s in the first-price auction with secret reserve R_0 does not have a closed-form solution. To implement the public reserve price R_0^P, we solve for $\beta_1^s(x)$ numerically. In the game indexed by $Y = 1$, the interim payoffs of bidder i with value x_i is

$$\begin{aligned} \pi(x_i; \widetilde{x}_i) &= (x_i \quad \beta_1^s(\widetilde{x}_i)) P(\beta_1^s(\widetilde{x}_i) \quad X_0 S \cap \beta_1^s(\widetilde{x}_i) \quad \beta_1^s(X_j), j \neq i) \\ &= (x_i \quad \beta_1^s(\widetilde{x}_i)) H(\beta_1^s(\widetilde{x}_i)) F_X(\widetilde{x}_i)^{n \quad 1} \end{aligned} \tag{14}$$

where H denotes the cumulative distribution of $X_0 S$. The first-order condition characterizing bidder i's truth-full bidding ($\widetilde{x}_i = x_i$, by the revelation principle) is

$$\frac{d\beta_1^s(x_i)}{dx} = (x_i \quad \beta_1^s(x_i)) \frac{(n \quad 1)\frac{f_X(x_i)}{F_X(x_i)} H(\beta_1^s(x_i))}{H(\beta_1^s(x_i)) \quad h(\beta_1^s(x_i))(x_i \quad \beta_1^s(x_i))}$$

or equivalently

$$\left[\frac{d}{dx} + (n-1)\frac{f_X(x_i)}{F_X(x_i)}\frac{1}{1-\frac{h(\beta_1^s(x_i))}{H(\beta_1^s(x_i))}(x_i-\beta_1^s(x_i))}\right]\beta_1^s(x_i) = x_i(n-1)\frac{f_X(x_i)}{F_X(x_i)}\frac{1}{1-\frac{h(\beta_1^s(x_i))}{H(\beta_1^s(x_i))}(x_i-\beta_1^s(x_i))}$$

with boundary conditions $\beta_1^s(\underline{x}) = \underline{x}$ and $\beta_1^s(\overline{x}) = \overline{b}$ for some unknown $\overline{b}$. We discretize the value space and solve the differential equation by fixed-point iteration (Fibich and Gavish (2011)). The algorithm performs well and converges to a unique fixed point from a large number of initial choices for the function $\beta_1^s(x_i)$. The corresponding inverse bid function ξ_1^s can be used to construct R_0^P.

D.2 Algorithm for solving the auction with learning rule (c)

In this section, we introduce the algorithm used to solve for the equilibrium of the first-price auction game in which the seller uses the learning rule given by Equation (7). Computation of an equilibrium is complicated by the fact that the distribution of the ex-post reserve price R_1 (from the perspective of bidder i) depends on the bidding strategy of rival firms.

Bidder i's interim payoff (Equation (8)) can be written

$$\begin{aligned}\pi(x_i, y; b_i) &= (x_i y - b)P(b_i \geq R_1(X_0, \widetilde{Y}, b_i, \mathbf{B}_{-i}; \gamma) \cap b_i \geq B_j, j \neq i \mid Y = y)\\ &= (x_i y - b)P(b_i \geq R_1(X_0, \widetilde{Y}, b_i, \mathbf{B}_{-i}; \gamma) \mid Y = y, b_i \geq B_j)P(b_i \geq B_j, j \neq i \mid Y = y)\end{aligned}$$

To win the auction, bidder i must outbid his rivals as well as the ex-post reserve price chosen by the seller. This reserve price is a function of the seller's ex-ante appraisal value and the vector of bids submitted. Denote by $W(b_i|Y = y) = P(b_i \geq R_1(X_0, \widetilde{Y}, b_i, \mathbf{B}_{-i}; \gamma) \mid Y = y, b_i \geq B_j)$ the probability that bidder i wins the item (i.e., submits a bid higher than the ex-post reserve price) given that he submitted the highest bid among all bidders. Note that this probability depends on the distribution or rivals' bids through $\mathbf{B}_{-i}$. Let β_y^c denoted an equilibrium of this game. Since the learning rule satisfies Assumption 1, Proposition 1 applies. We have that $\beta_y^c = y \cdot \beta_1^c$.

The first-order condition characterizing bidder i's equilibrium bid function, in the game indexed by $Y = 1$, is

$$\frac{d\beta_1^c(x_i)}{dx} = (x_i - \beta_1^c(x_i))\frac{(n-1)\frac{f_X(x_i)}{F_X(x_i)}W(\beta_1^c(x_i)|Y=1)}{W(\beta_1^c(x_i)|Y=1) - w(\beta_1^c(x_i)|Y=1)(x_i - \beta_1^c(x_i))} \tag{15}$$

with boundary conditions $\beta_1^c(\underline{x}) = \underline{x}$. The solution, for a given value of the parameter γ, is found numerically by best-response iteration using Algorithm 1 (in particular, in each iteration, the non-linear problem Equation (15) is solved by an inner fixed-point iteration method). Following this approach, the learning rule is updated at each iteration.

Algorithm 1 Equilibrium solver

Initialize the bid function $\beta_1^c(.)$ (e.g., $\beta_1^c(x) = x$)
$\Delta := \epsilon + 1$
while $\Delta > \epsilon$ **do**
    Define the learning rule as in Equation (7), given $\beta_1^c(.)$.
    Compute the probability of winning with a bid b if rivals follow strategy $\beta_1^c(.)$

$$P(b \geq R_1(X_0, S, b, \mathbf{B}_{-i}; \gamma) \cap b \geq B_j, j \neq i | Y = 1)$$

    Find the best-response against strategy $\beta_1^c(.)$ (solving ODE Equation (15) via fixed-point iteration)

$$\beta_1(x_i) = \arg\max_b (x_i - b) P(b \geq R_1(X_0, S, b, \mathbf{B}_{-i}; \gamma) \cap b \geq B_j, j \neq i | Y = 1)$$

    where $\mathbf{B}_{-i}$ are rival bids generated using strategy $\beta_1^c(.)$
    Update the best-response: $\beta_1^c := \beta_1$
    $\Delta := ||\beta_1 - \beta_1^c||_2$
end while

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